Efficiency in Heterogeneous Firm Trade Models

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To assess the optimality of market allocations resulting from international trade, we need to clarify the policymaker's objective function over different international pairings between producers and consumers. This is because every linkage between a producer in country j and a consumer in country i may encounter trade frictions distinct from one another, and a policymaker will factor the costs of each linkage in their decisions. We define social welfare W over allocations of goods $\{Q_{ji}\}$ produced in j and sold in country i to a worker k as

$$W\left(\{Q_{ji}\}\right) \equiv \int_{k \text{ is a worker}} \min_{i,j} \{U(Q_{ji})/\omega_{ji}\}\, dk \tag{1}$$

where U is each worker's utility and $\omega_{ji} > 0$ is the Pareto weight for country i's consumption of goods from j.

In our setting, workers are treated identically by producers within each country. Accordingly, we constrain the social planner to provide the same allocation to all workers within a country. We identify each worker i with her country I and a country-wide Pareto weight ω_{JI} which weights utility from goods produced in J. Each country has a mass L_I of workers, which allows us to aggregate within each country and write social welfare as

$$W = \sum_{I \text{ is a country}} L_I \min_{I,J} \{U(Q_{JI})/\omega_{JI}\} = \min_{I,J} \{U(Q_{JI})/\omega_{JI}\} \cdot \sum_I L_I. \tag{2}$$

From Equation (2), dividing both sides by the world population shows any socially optimal allocation maximizes per capita welfare, using appropriate Pareto weights for each country pairing (J,I).[1] For any Pareto efficient allocation $\{Q^*_{JI}\}$, defining weights so that $\omega_{JI}/\omega_{J'I'} =$

[1] Our specification of social welfare is consistent with the trade agreement literature. Bagwell and Staiger (2009) focus on equal weights as home and foreign labor are directly comparable in their model due to the presence of an outside homogeneous good.

$U(Q^*_{JI})/U(Q^*_{J'I'})$ shows $\{Q^*_{JI}\}$ must maximize W (otherwise a Pareto improvement is possible). Since every Pareto efficient allocation corresponds to some set of weights $\{\omega_{ji}\}$, ranging over all admissible weights $\{\omega_{JI}\}$ sweeps out the Pareto frontier of allocations in which there is a representative worker for each country. Thus, any market allocation can be evaluated for Pareto efficiency in the usual way using Equation (2).

Proposition. *Every market equilibrium of identical open Melitz economies with trade frictions is socially optimal.*

Proof. Following the discussion of social welfare under trade, we will show that the market allocation is Pareto efficient. Concretely, the products that j produces and are consumed by i are a triple $Q_{ji} = \left(M_e^{ji}, c_d^{ji}, q_{ji}\right)$ which provides welfare of $U\left(Q_{ji}\right) \equiv M_e^{ji} L_i \int_0^{c_d^{ji}} \left(q_{ji}(c)\right)^\rho g(c)dc$. As laid out in the definition of social welfare, these j and i are representative, and the optimal allocation is one that maximizes $W \equiv \min_{i,j}\left\{U(Q_{ji})/\omega_{ji}\right\}$ for some Pareto weights $\left\{\omega_{ji}\right\}$. Since labor is not mobile and resources are symmetric ($L_j = L$ for all j), one can maximize W by considering the goods produced by each country j separately. Accordingly, fix $j = 1$ so maximizing W amounts to maximizing

$$W^1 \equiv \min_i \left\{U(Q_{1i})/\omega_{1i}\right\}. \tag{3}$$

Since U is increasing (if every element of a product vector Q' is strictly greater than a product vector Q then $U(Q') > U(Q)$) it is easy to see that any $\left\{Q^*_{1i}\right\}$ that maximizes W^1 is characterized exactly by simultaneously being on the Pareto frontier while $U(Q_{1i})/U(Q_{1j}) = \omega_{1i}/\omega_{1j}$. Since Equation (3) is difficult to deal with directly, we will now maximize an additive social welfare function $\mathscr{W}^1 \equiv U(Q_{11}) + \sum_{j>1} U(Q_{1j})$. This is because any allocation which maximizes $\mathscr{W}^1$ must be Pareto efficient, as any Pareto improvement increases $\mathscr{W}^1$. Since the Pareto weights are free, at any maximum $\left\{Q^*_{1i}\right\}$ we may set $\omega_{1i} \equiv U(Q^*_{1i})$ so that $\left\{Q^*_{1i}\right\}$ maximizes Equation (3).

$\mathscr{W}^1$ must be maximized subject to a joint cost function $C\left(\{Q_{1i}\}\right)$ we now detail. For brevity define the two "max" terms $\overline{M} \equiv \max_j\{M_e^{1j}\}$ and $\overline{c} \equiv \max_j\{c_d^{1j}\}$ and the "fixed" cost function $C_f\left(\overline{M}, \overline{c}\right) \equiv \overline{M}\left(f_e + G(\overline{c})f\right)$ which is incurred from fixed costs at home. Next define "variable" costs at home $C_1\left(Q_{11}\right)$ and abroad $C_j\left(Q_{1j}\right)$ by

$$C_1 \equiv M_e^{11} L \int_0^{c_d^{11}} c q_{11}(c) g(c) dc \quad \text{and} \quad C_j \equiv M_e^{1j} \int_0^{c_d^{1j}} \left(L\tau c q_{1j}(c) + f_x\right) g(c) dc$$

where $\tau = \tau_{ji}$ denotes the symmetric transport cost. Then total costs are given by $C\left(\{Q_{1i}\}\right) = C_f\left(\overline{M}, \overline{c}\right) + C_1\left(Q_{11}\right) + \sum_{j>1} C_j\left(Q_{1j}\right)$.

Now fix $\left\{M_e^{1j}\right\}$ and $\left\{c_d^{1j}\right\}$ which fixes C_f. Also fix some allocation of labor across variable costs, say $\left\{\mathscr{L}_j\right\}$, with $C_f + \sum \mathscr{L}_j = L$, that constrain $C_j \leq \mathscr{L}_j$. We may then maximize each $U(Q_{1j})$ subject to the constraint $C_j \leq \mathscr{L}_j$ separately and we may assume WLOG that each $\mathscr{L}_j > 0$.[2] As in the argument for the closed economy, sufficient conditions for maximization with $\left\{M_e^{1j}\right\}$ and $\left\{c_d^{1j}\right\}$ fixed are

$$q_{11}^*(c) = c^{1/(\rho-1)} \mathscr{L}_1 / M_e^{11} L R(c_d^{11}), \tag{4}$$

$$q_{1j}^*(c) = c^{1/(\rho-1)} [\mathscr{L}_j / M_e^{1j} - f_x G(c_d^{1j})] / L R(c_d^{1j}) \tau. \tag{5}$$

Having found the optimal quantities of Equations (4-5) in terms of finite dimensional variables, we now prove existence of an optimal allocation. Note that for any fixed pair $(\overline{M}, \overline{c})$, the remaining choice variables are restricted to a compact set $K(\overline{M}, \overline{c})$ so that continuity of the objective function (by defining $U(Q_{1j}) = 0$ when $\mathscr{L}_j = 0$) guarantees existence of a solution and we denote the value of $\mathscr{W}^1$ at the maximum by $S(\overline{M}, \overline{c})$. In fact, $K(\overline{M}, \overline{c})$ can be shown to be a continuous correspondence, so by the Theorem of the Maximum $S(\overline{M}, \overline{c})$ is continuous on $C_f^{-1}([0, L])$ (Berge and Karreman, 1963). Since C_f is continuous, $C_f^{-1}([0, L])$ is compact and therefore a global max of $S(\overline{M}, \overline{c})$ exists. Therefore there is an allocation that maximizes $\mathscr{W}^1$ which we now proceed to characterize.

Now evaluating welfare at the quantities of Equations (4-5) yield respectively

$$U(Q_{11}) = R(c_d^{11})^{1-\rho} L^{1-\rho} M_e^{11} \left(\mathscr{L}_1 / M_e^{11}\right)^{\rho}, \tag{6}$$

$$U(Q_{1j}) = R(c_d^{1j})^{1-\rho} L^{1-\rho} M_e^{1j} \left(\mathscr{L}_j / M_e^{1j} - f_x G(c_d^{1j})\right)^{\rho} \tau^{-\rho}. \tag{7}$$

Equation (6) is increasing in both M_e^{11} and c_d^{11} so it follows that at any optimum, $M_e^{11*} = \overline{M}$ and $c_d^{11*} = \overline{c}$. Equation (7) is first increasing in M_e^{1j}, attains a critical point at $(1-\rho)\mathscr{L}_j / f_x G(c_d^{1j})$ and is then decreasing, so optimal $M_e^{1j*} = \min\left\{(1-\rho)\mathscr{L}_j / f_x G(c_d^{1j}), \overline{M}\right\}$. If $c_d^{1j*} < \overline{c}$ then the first order necessary condition implies

$$M_e^{1j} = (1-\rho)\mathscr{L}_j / f_x \left(\rho R(c_d^{1j}) / \left(c_d^{1j}\right)^{\rho/(\rho-1)} + (1-\rho) G(c_d^{1j})\right) < (1-\rho)\mathscr{L}_j / f_x G(c_d^{1j})$$

so $c_d^{1j*} < \overline{c}$ implies $M_e^{1j*} = \overline{M}$ and $M_e^{1j*} < \overline{M}$ implies $c_d^{1j*} = \overline{c}$. Ruling out the latter case, $M_e^{1j*} <$

[2] If $\mathscr{L}_j = 0$ for all j then autarkic allocations are optimal, and as shown above the optimal autarkic allocation coincides with the market. Any set of exogenous parameters which result in trade imply welfare beyond autarky, so if countries trade in the market equilibrium, $\mathscr{L}_j = 0$ for all j cannot be optimal. Inada type conditions on $U(Q_{1j})$ imply that if it is optimal to have at least one $\mathscr{L}_j > 0$ then all $\mathscr{L}_j$ are > 0.

$\overline{M}$ implies $U(Q_{1j}) = \tau^{-\rho} L^{1-\rho} (1-\rho)^{1-\rho} \rho^\rho \mathscr{L}_j f_x^{\rho-1} \left(R(c_d^{1j})/G(c_d^{1j})\right)^{1-\rho}$ which is decreasing in c_d^{1j} so $c_d^{1j*} = \overline{c}$ cannot be optimal. Therefore we conclude that $M_e^{1j*} = \overline{M}$ and $c_d^{1j*} < \overline{c}$. In particular, c_d^{1j*} must solve the implicit equation

$$\rho R(c_d^{1j*}) / \left(c_d^{1j*}\right)^{\rho/(\rho-1)} + (1-\rho)\, G(c_d^{1j*}) = (1-\rho)\, \mathscr{L}_j / \overline{M} f_x \tag{8}$$

derived from the first order necessary condition.

With these results in hand, $\mathscr{W}^1$ reduces to

$$\mathscr{W}^1 = \left(\overline{M}L\right)^{1-\rho} \left\{ R(\overline{c})^{1-\rho} \mathscr{L}_1^\rho + \tau^{-\rho} \sum_{j>1} R(c_d^{1j})^{1-\rho} \left(\mathscr{L}_j - \overline{M} f_x G(c_d^{1j})\right)^\rho \right\}. \tag{9}$$

Now consider maximizing $\mathscr{W}^1$ as given in Equation (9) over $\overline{M}, \overline{c}, \mathscr{L}_j, c_d^{1j}$ with c_d^{1j} unconstrained by $\overline{c}$ for $j > 1$. Using a standard Lagrangian approach, the candidate solution from the necessary conditions implies $c_d^{1j*} = (f_x/f)^{(\rho-1)/\rho} \overline{c}/\tau$ and since it is assumed $(f/f_x)^{(1-\rho)/\rho} < \tau$ for trade in a market equilibrium in the Melitz framework, $c_d^{1j*} < \overline{c}$. The candidate solution with c_d^{1j} unconstrained also yields Equation (8) so the unconstrained candidate solution coincides with the solution including the omitted constraints $c_d^{1j*} < \overline{c}$. We conclude the necessary conditions embodied in the candidate solution are also necessary to maximize $\mathscr{W}^1$ with constraints. Since these necessary conditions are exactly those which fix the unique market allocation, the market allocation maximizes $\mathscr{W}^1$. □

Proposition shows the market allocation is efficient in the presence of trade costs. We note however that the variable trade costs must be ad valorem for this result. Specific trade costs produce distortions because firms choose markups that are not proportionate to the marginal utility at the optimal allocation.

References

Bagwell, K. and R. W. Staiger, "Delocation and trade agreements in imperfectly competitive markets," *NBER Working Paper*, 2009.

Berge, Claude and Karreman, *Topological spaces, including a treatment of multi-valued functions, vector spaces and convexity*, New York: Macmillan, 1963.