

# Trading Away Wide Brands for Cheap Brands

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This draft: November 15, 2012

## **Abstract**

Firms face competing needs to expand product variety and reduce production costs. Access to larger markets enables innovation to reduce costs. Although firm scale increases, foreign competition reduces markups. Firms' ability to recapture lost markups depends on the interplay between within-firm competition and across-firm competition. Narrowing product variety eases within-firm competition but lowers market share. I provide a theory detailing the impact of trade policy on product and process innovation. Unbundling innovation provides new insights into welfare gains and innovation policy. Product innovation increases welfare beyond standard gains from trade. The relative returns to innovation policy change with trade liberalization.

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*Acknowledgment.* I am grateful to Bob Staiger for continued guidance and to Charles Engel and Steven Durlauf for their support and comments. This paper has benefited from helpful comments of David Atkin, Maria Bas, Andy Bernard, South Bisonyabut, Stefania Garetto, Scott Gehlbach, Elhanan Helpman, Wolfgang Keller, Amit Khandelwal, Rasmus Lentz, Rebecca Lessem, Jim Lin, Kalina Manova, Marc Melitz, Marc Muendler, Peter Neary, Nina Pavcnik, Dan Quint, Natalia Ramondo, Steve Redding, Esteban Rossi-Hansberg, Scott Taylor, Stephen Yeaple, Alwyn Young, Mian Zhu, participants at various seminars, two anonymous referees and especially John Morrow.

# 1 Introduction

Trade liberalization provides welfare gains by increasing product variety and productivity within industries. Krugman (1980) and Melitz (2003) show how trade provides these gains from entry and exit of firms. Empirical studies confirm the importance of this channel of entry and exit. At the same time, these studies point to the contribution of a second channel: innovation responses within firms. Firms make investments to increase their product variety and lower their production costs. These innovation activities make up a large fraction of aggregate changes in industry-level variety and productivity.<sup>1</sup> This paper examines how trade policy affects firm investments in product variety and cost reduction.

Differences in product and process innovation are empirically important in shaping relevant market outcomes such as product life cycles, firm growth, industry evolution and export participation.<sup>2</sup> Standard models explain how trade liberalization encourages process innovation but do not address the tradeoff between product and process innovation. In these models, innovation occurs only along the process dimension and product variety is exogenously fixed within firms. Trade expands market size and enables firms to exploit economies of scale in process innovation. With a better production process, firms can produce a higher quantity (or better “quality”) at the original production cost. However, opening to trade has no effect on product innovation and the question of how trade policy affects firms’ investment tradeoffs cannot be addressed.

The answer to this question must hinge on how product and process choices differ from each other. If the net returns to these choices are qualitatively similar, then looking at aggregate innovation will suffice for many questions of interest. However, as observed earlier, differences in product and process innovation matter for market outcomes and modeling these differences could provide new insights into the impact of trade policy. This paper provides a theory detailing differences between product and process innovation and how they matter for the impact of trade liberalization on welfare and innovation policy. Firms invest in product variety and production processes, and I show that unbundling innovation reveals welfare gains from trade which do not arise in the absence of a distinction between product and process innovation. Further, trade liberalization affects the relative benefits from product and process innovation and reveals new tradeoffs in innovation policy.

To model innovation, I focus on demand side effects of introducing new products and processes. This gives an explanation for why product and process innovation differ and how trade

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<sup>1</sup>For example, Bernard et al. (2010) find that within-firm product expansion accounts for about half of US output of new products. Doraszelski and Jaumandreu (2007) show within-firm productivity growth accounts for two-thirds of total productivity gains among Spanish firms.

<sup>2</sup>For example, Abernathy and Utterback (1978), Klepper (1996) and Becker and Egger (2007) document these effects.

affects them through different channels. To formalize the distinctions, I model brand differentiation in a monopolistic competition setting. Consumers have a taste for diversity in brands and demand products from different brands. Each firm makes a unique brand of products, and following the marketing literature, branding enables a firm to differentiate its products from those of its competitors (Aaker 1991, Chapter 1). Within its unique brand, a firm makes multiple products. Introducing a new product widens the brand and enables the firm to amortize the sunk costs of establishing its brand. At the same time, a new product lowers the existing market share of the brand as consumers substitute into the brand's new product. For instance, when Yoplait introduces a new yogurt, demand for its original yogurt falls. This within-brand cannibalization induces a natural distinction between the returns to product and process innovation. When a firm widens its product variety, market shares of its existing products are cannibalized. In contrast, upgrading the production process of a product reduces its unit cost without cannibalizing existing market shares. Process innovation therefore reflects economies of scale in the usual way; as quantity of a product rises, investments in its production process become more profitable.

These two channels of economies of scale and cannibalization together explain why trade can have different effects on the returns to product and process innovation. I start with the assumption that consumers consider products to be more substitutable within brands than across brands.<sup>3</sup> For example, when Yoplait introduces a new yogurt, demand for its original yogurt falls more than demand for an original Dannon yogurt. In this benchmark model, moving from autarky to free trade increases market size and raises the returns to process innovation through economies of scale. At the same time, opening to trade introduces import competition from foreign brands and lowers the residual demand for each domestic brand. Facing within-brand cannibalization, each firm recognizes that it can cope with external competition from imports by cutting back on internal competition within its own brand. As a consequence, trade induces firms to lower product innovation (through cannibalization) but increase process innovation (through economies of scale).

These firm responses have conflicting implications for welfare gains from trade. Process innovation raises productivity and increases welfare through lower prices. The drop in product innovation lowers welfare from domestic variety. This welfare loss is overcome by access to foreign brands and consumers experience welfare gains from increase in total variety. In the presence of brand differentiation, welfare from variety rises for another reason. Wide brands give way to narrow brands and consumers get access to more differentiated varieties. A direct observation is that this gain would not arise in a model with no brand differentiation. A less immediate result is that consumers would not get this gain if firms are unable to divert their investments into

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<sup>3</sup>Empirical evidence for higher substitutability within brands is provided by Broda and Weinstein (2010) using supermarket data and Hui (2004) using personal computers data. Also see Hui for a summary of supporting business and consumer psychology theories.

process innovation. Process innovation enables firms to take advantage of market expansion without increasing product lines. Innovation therefore provides welfare gains from lower prices and access to more differentiated products. This has implications for how innovation policy can be targeted in response to lower trade barriers. Opening to trade changes the relative social benefits from product and process innovation. As market size expands, the complementarity between scale and process innovation becomes more important. The benefits from process innovation rise and opening to trade makes inadequate process innovation more costly. Innovation policies routinely require information on investment projects and this finding shows a greater need to support process investments over product innovation after trade liberalization.

These results explain the impact of trade on innovation when within-brand cannibalization is higher than across-brand competition. But they miss out on meaningful tradeoffs that arise when product introductions affect across-brand competition. As a firm widens its brand, it might crowd the product space and intensify across-brand competition. On the other hand, widening of a brand might increase visibility to consumers and hence lower the across-brand competition faced by the brand. Industry studies show both effects are plausible.<sup>4</sup> I therefore incorporate richer substitutability patterns and examine how trade affects welfare gains and innovation policy across markets with different demand characteristics. To understand the interplay between within-brand and across-brand competition, I allow demand for a variety to depend on purchases of the variety's own brand, purchases of similar products across all brands and an interaction between brand-level and product-level purchases. Within-brand substitutability need not be larger than across-brand substitutability, and brand widening can affect across-brand competition in different ways. A negative interaction implies brand expansion intensifies across-brand competition. This sharpens the rise in demand elasticities after trade and puts further pressure on firms to cut back on product lines. A positive interaction implies brand expansion increases visibility to consumers. In markets with high search costs, consumers are expected to place a high value on visibility of varieties and hence prefer bigger brands.<sup>5</sup> In these markets, firms have an incentive to increase product innovation despite higher cannibalization after trade.

This increase in product innovation provides welfare gains from greater total variety. At the same time, it gives consumers access to lower elasticity varieties. As foreign brands enter, the returns to lowering across-brand competition rise. The positive demand effect from visibility is

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<sup>4</sup>For example, Akerberg and Rysman (2005) find substantial crowding in the yellow pages industry and discuss the role of product space overlap in demand estimation. On the other hand, Gavazza (2011) finds consumers concentrate their purchases towards wider brands and positive demand spillovers accrue to wider brands in the retail mutual funds industry. Still differently, Gowrisankaran and Rysman (2009) estimate negligible demand spillovers from new products in the digital camcorder industry.

<sup>5</sup>For example, Bronnenberg et al. (forthcoming) find migrants switch 60 per cent of their original budget towards the largest brand of their new home state. See Nevo (2000) for a broad overview of branding and product characteristics in demand estimation.

higher and within-brand cross-elasticities fall after trade liberalization. This provides welfare gains from product innovation. As consumers value bigger brands, the benefits from product innovation are higher than the benefits from entry of new brands, so trade liberalization reduces the need to encourage entry of brands. However, the need to support process innovation remains. Firms internalize the benefits from product innovation but do not fully account for the improved opportunities to expand production through process innovation.

These results provide testable predictions for how trade affects product and process innovation, depending on the nature of within-brand and across-brand competition across markets. As finer firm-level data is becoming available, I move beyond predictions for average innovation levels and incorporate firm heterogeneity to obtain richer testable predictions. Under firm heterogeneity in initial productivity, the impact of trade policy on market expansion and competition is similar to that with homogeneous firms. But now each firm has a different share of the domestic and export markets so innovation responses vary across firms. A bilateral tariff reduction expands the market size available to new and continuing exporters, leading to a rise in process innovation. Continuing non-exporters experience no change in market size and hence have no additional incentive to engage in process innovation. However, they face import competition from entry of foreign brands and cut back on product lines. High-productivity exporters capture the largest gains from increasing market size and are able to overcome tougher competition to increase product lines. They engage in higher product and process innovation after a bilateral reduction in tariffs. A unilateral home tariff liberalization induces the opposite firm responses because it shifts market shares away from home firms.

The main contribution of this paper is to systematically examine how trade policy affects different dimensions of innovation. In contrast to the classic work of Grossman and Helpman (1991), I show that product and process innovation have varying implications for both positive and normative questions such as differences in firm responses and the welfare impact of trade and innovation policy.<sup>6</sup> Aggregate innovation conceals these qualitative differences. I integrate insights from work on innovation and multiproduct firms to address these differences. The innovation literature underlines how trade liberalization affects aggregate innovation through economies of scale (Grossman and Helpman 1993). Yeaple (2005), Bustos (2009) and Lileeva and Trefler (2010) show higher scale from exports is positively associated with better technologies. The important findings of Bustos and Lileeva and Trefler provide evidence for this positive relationship among Argentinean and Canadian firms. I consider these scale economies and introduce demand linkages to address the tradeoff between technology and product variety.

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<sup>6</sup>Grossman and Helpman consider separate models with quality and variety. They find that both models yield the same positive results. However, there are differences in normative results. Variety models show firms do too little innovation (due to intertemporal spillovers) but quality models may have too little or too much innovation.

To model product innovation, I build on recent work on multiproduct firms in international trade. Eckel and Neary (2010), Feenstra and Ma (2007) and Ju (2003) study cannibalization arising from strategic across-firm competition among oligopolistic firms. I consider within-brand cannibalization which is a complementary feature as it can co-exist with strategic across-firm competition in oligopolistic industries.<sup>7</sup> Within the monopolistic competition setting, Arkolakis and Muendler (2007), Bernard et al. (2010, 2011) and Mayer et al. (2009) study the role of cost linkages within multiproduct firms. They propose that increasing product lines entails higher production or market penetration costs, leading to differences in products sold across markets.<sup>8</sup> I abstract from differences in products across markets as my main purpose is to address product and process innovation. Instead, I study demand linkages within firms and show how they drive a wedge between the returns to product and process innovation. As in these papers, I focus on trade liberalization in final goods and abstract from other aspects of globalization such as tariff reductions on technology and intermediate goods. Goldberg et al. (2009) find lower tariffs on intermediate inputs increase firms' incentives to introduce new products. While I do not explicitly model imports of inputs, the innovation policy results illustrate how availability of cheaper intermediate inputs and technology goods increases product and process innovation. This implies innovation responses could differ when a unilateral home tariff liberalization includes reductions in input tariffs.

The paper is organized as follows. Section I introduces brand differentiation and examines the relationship between cannibalization, innovation and trade liberalization. In Section II, I consider richer substitutability patterns to examine how trade affects innovation, welfare and innovation policy across different markets. Section III introduces firm heterogeneity to provide testable predictions for innovation responses across firms. Section IV concludes.

## **2 Benchmark Model: Within-Brand Cannibalization and Innovation**

This Section provides a model of multiproduct firms that invest in product variety and production processes. I start with the simplest model that yields two main results. First, returns to product and process innovation differ substantively through within-brand cannibalization. Second, trade affects product and process innovation through different channels.

To model these distinctions, I propose a linear demand structure with brand differentiation. The standard approach to model brand differentiation is through nested CES preferences where

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<sup>7</sup>Unlike oligopolistic multiproduct models with fixed number of firms, I study monopolistically competitive firms with free entry. Entry and exit of firms play a crucial role in welfare gains in my model.

<sup>8</sup>Eckel and Neary (2010) and Nocke and Yeaple (2005) propose rising production costs as well. Bernard et al. (2010, 2011) also consider the role of differences in product attributes in determining product lines.

the first nest is defined over brands and the second nest over multiple products within each brand.<sup>9</sup> I depart from the standard CES assumption because it has very special implications for the relation between trade and innovation. In an online Appendix, I show that nested CES preferences imply trade liberalization has no effect on product and process innovation in Krugman (1980) and on process innovation in Melitz (2003). As is well-known, CES preferences are special in inducing all adjustments through the extensive margin of product variety. Product market competition and the rate of cannibalization are exogenously fixed so the returns to innovation are not altered through this channel. Consequently, I provide a linear demand model where product market competition and the rate of cannibalization vary with trade liberalization, leading to differential effects on returns to innovation. I start with an exposition of the closed economy and then study the effects of trade liberalization.

Consider a closed economy with  $L$  identical agents, each endowed with a unit of labor. Total income in the economy is  $I = wL$  where  $w$  is the wage rate (normalized to 1). Agents work in one of two industries: a homogeneous goods industry or a differentiated goods industry. In the homogeneous goods industry, producers are perfectly competitive and produce under constant returns to scale with a unit labor requirement. In the differentiated goods industry, firms are monopolistically competitive. They pay an entry cost  $f$  to enter and produce a brand of goods. Firms can produce multiple products within a brand. I explain the role of brands in the following subsection and then consider its implications for product and process innovation.

## 2.1 Demand

Agents in the home country have identical preferences defined over a homogeneous and a differentiated good. Agent  $k$  consumes  $q_0^k$  of the homogeneous good and  $q_{ij}^k$  of variety  $i \in \Omega_j$  of brand  $j \in \mathcal{J}$  of the differentiated good. Her total consumption of brand  $j$  goods is  $q_j^k \equiv \int_i q_{ij}^k di$ . Her aggregate consumption of differentiated goods of all brands is  $Q^k \equiv \int_j q_j^k dj$ . Agent  $k$  derives the following utility from her consumption of homogeneous and differentiated goods:

$$U^k \equiv q_0^k + \alpha Q^k - \frac{\delta}{2} \int_j \int_i (q_{ij}^k)^2 didj - \frac{\gamma}{2} \int_j (q_j^k)^2 dj - \frac{\eta}{2} (Q^k)^2. \quad (2.1)$$

Parameters  $\alpha$ ,  $\delta$ ,  $\gamma$  and  $\eta$  are all strictly positive. As in Melitz and Ottaviano (2008),  $\alpha$  and  $\eta$  determine substitutability between the homogeneous and differentiated goods. Parameter  $\delta$  captures the degree of differentiation across varieties. Lower  $\delta$  implies varieties are less differentiated and hence more substitutable with  $\delta = 0$  denoting consumers have no taste for diversity in varieties.

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<sup>9</sup>Allanson and Montagna (2005) use a nested CES demand structure to examine product variety within firms. The production side of this economy is similar to Krugman (1980) so it can be readily extended to study the impact of trade on product and process innovation.

Unlike Melitz and Ottaviano,  $\gamma$  captures the degree of differentiation across brands with  $\gamma = 0$  implying no brand differentiation. This is a novel feature of the preference structure which I discuss in detail below.

In an equilibrium where agent  $k$  consumes both homogeneous and differentiated goods, the inverse demand function is

$$p_{ij} = \alpha - \delta q_{ij}^k - \gamma q_j^k - \eta Q^k. \quad (2.2)$$

From Equation (2.2), variety-level consumption  $q_{ij}^k$ , brand-level consumption  $q_j^k$  and market-level consumption  $Q^k$  can have different effects on the inverse demand. As in Melitz and Ottaviano, the inverse demand falls more with a rise in variety-level consumption than market-level consumption. Equation (2.2) shows brand-level consumption and market-level consumption can also have different effects on the inverse demand. Melitz and Ottaviano consider the case where  $\gamma = 0$  so varieties belonging to the same brand and varieties of other brands have identical effects on the inverse demand. Grouping of varieties by brands has no effect on willingness to pay and consumers of an original Yoplait yogurt are indifferent between consuming a new Yoplait yogurt or a new Dannon yogurt. Following the marketing and industrial organization literature, I define a brand as a set of varieties with demand linkages. Varieties of the same brand are substitutable for each other ( $\gamma > 0$ ) and consumers' willingness to pay for a variety falls as she consumes more varieties from the same brand. I refer to this fall in willingness to pay (due to consumption of the same brand) as within-brand cannibalization.

Under the inverse demand of Equation (2.2),  $\gamma > 0$  implies willingness to pay falls more with a rise in consumption of varieties belonging to the same brand rather than varieties of other brands. I will relax this assumption later but for now an increase in consumption of the original Yoplait yogurt reduces demand for a new Yoplait more than demand for a new Dannon yogurt. Consumers consider products to be more substitutable within brands than across brands. This can be expressed more clearly in terms of cross-elasticities of demand. Let  $q_{ij}$  be the total demand for variety  $i$  of brand  $j$  across all agents. With identical agents, each agent  $k$  demands  $q_{ij}^k = q_{ij}/L$ . Substituting for  $q_{ij}^k$ , total demand for variety  $i$  of brand  $j$  is  $q_{ij} = (L/\delta)[\alpha - p_{ij} - \gamma q_j/L - \eta Q/L]$  where  $q_j \equiv Lq_j^k$  and  $Q \equiv LQ^k$ . Cross elasticity of variety  $ij$  with respect to any other variety  $i'j'$  is  $\varepsilon_{ij,i'j'} \equiv -(dq_{ij}/dq_{i'j'})(q_{i'j'}/q_{ij}) = (1_{j=j'}\gamma + \eta)(q_{i'j'}/\delta q_{ij})$  and within-brand cross elasticity exceeds across-brand cross elasticity. The elasticities vary with quantities and branding, and this has implications for firm decisions which are discussed in the next sub-section.

## 2.2 Firms

Having explained within-brand cross-elasticities, I examine its implications for product and process innovation. I first explain firm decisions in a setting of symmetric firms under autarky and

then discuss the effects of trade.

In the differentiated goods industry, firms enter freely by paying a cost  $f$ . After paying entry costs, they can make products within a brand at a unit cost  $c$ . Firms have perfect information of the unit cost before paying entry costs. Having paid the entry cost, each firm faces three choices: which production process to use, what quantity to produce and how many products to supply. Firm  $j$  can either make product  $i$  at unit cost  $c$  or choose a lower unit cost  $c(\omega_{ij})$  by investing in process  $\omega_{ij}$ . I assume  $c(\omega_{ij}) \equiv c - c\omega_{ij}^{1/2}$  for  $\omega_{ij} \in [0, 1]$ . Higher levels of  $\omega_{ij}$  correspond to lower levels of unit cost ( $c'(\omega_{ij}) < 0$ ) with  $c(0) = c$  denoting no process innovation and  $c''(\omega_{ij}) > 0$ .<sup>10</sup> Upgrading to process  $\omega_{ij}$  entails expenditure on technology adoption or investment in process R&D at a rate  $r_\omega$ . Firm  $j$  chooses how much of product  $i$  to supply to the home market ( $q_{ij}$ ). It chooses this quantity faced with an inverse demand  $p_{ij} = (\alpha - \eta Q/L) - \delta q_{ij}/L - \gamma q_j/L \equiv a - \delta q_{ij}/L - \gamma q_j/L$ . The inverse demand intercept  $a \equiv \alpha - \eta Q/L$  summarizes market demand conditions that firm  $j$  takes as given. Firm  $j$  can make multiple products to amortize its entry costs. It chooses a product range of  $h_j$  products by investing in product R&D at a rate  $r_h$  per product.<sup>11</sup>

Putting these choices together, firm  $j$  decides on its production process  $\omega_{ij}$  and quantities  $q_{ij}$  for each product  $i$  along with its product range  $h_j$  to maximize the following profit function.

$$\max_{\omega_{ij}, q_{ij}, h_j} \Pi_j \equiv \int_0^{h_j} \{ [p_{ij} - c(\omega_{ij})] q_{ij} - r_\omega \omega_{ij} - r_h \} di - f.$$

Firms face no uncertainty of costs or payoffs and no new information is revealed at any stage. As a result, the sequencing of firm decisions does not matter. With symmetric costs within firms, firm  $j$  chooses the same process and quantities for each product supplied and the firm-product subscripts can be suppressed.<sup>12</sup> The firm problem can be re-written as  $\Pi = h\{ [p - c(\omega)]q - r_\omega \omega - r_h \} - f \equiv h\pi - f$  where  $\pi$  is profit per product. In what follows, I determine the optimal production process  $\omega$ , quantity  $q$  and product range  $h$  through FOCs for the firm problem.

<sup>10</sup>The specific functional form for  $c(\omega)$  is not crucial. Results are similar as long as the firm problem is concave. Sufficient conditions for an interior equilibrium are in the Appendix. A detailed proof is available in an online Appendix.

<sup>11</sup>The view of process innovation as vertical differentiation (more quantity per unit cost) and product innovation as horizontal differentiation is similar to Eswaran and Gallini (1996). In this view, firms may increase or decrease investments in product and process innovation. Product innovation refers to a rise in product variety and not to improvements in product “quality” (which yields more utility-effective quantity per unit cost). For a related literature on quality, the reader may refer to Kugler and Verhoogen (2012), Hallak and Sivadasan (2006) and Eckel et al. (2009).

<sup>12</sup>I allow firms to choose a production process for each product. Results are similar for intermediate levels of product-specificity of process R&D costs. In the extreme case of costless application of the production process to all products of the firm, my model collapses to the standard case where product and process innovation only vary on the cost side and not the demand side.

### 2.2.1 Production Process

The FOC for process choice is  $\partial\pi/\partial\omega = -c'(\omega)q - r_\omega = 0$ . A firm invests in process R&D until savings from lower unit costs (net of the process R&D cost) are driven to zero. Two points are worth mentioning. First, process innovation  $\omega$  reflects economies of scale through  $q$ . As scale per product rises, process innovation becomes more profitable. Second, process innovation does not directly cannibalize. Given its other decisions ( $q$  in this case), this firm would have chosen the same process in the absence of cannibalization (when  $\gamma = 0$ ). Later I show that process innovation does not cannibalize even after accounting for equilibrium quantity.

### 2.2.2 Quantity

With symmetric quantities, total supply of firm  $j$  is  $q_j = \int_i q_{ij} di = hq$ . This implies the inverse demand is  $p = a - \delta q/L - \gamma hq/L$ . Quantity  $q$  lowers consumers' willingness to pay through its own effect ( $\delta q/L$ ) and its brand-level effect ( $\gamma hq/L$ ). The FOC for quantity supplied to the domestic market is

$$\partial\pi/\partial q = \underbrace{[p - (\delta + \gamma h)q/L]}_{\text{MR}} - \underbrace{c(\omega)}_{\text{MC}} = 0. \quad (2.3)$$

The marginal revenue includes  $\gamma hq/L$  and shows branded multiproduct firms reduce their quantities in anticipation of cannibalization of old products. Figure 2.1 illustrates the optimal quantity choice. The x-axis reports quantities while the y-axis reports prices, marginal revenue and unit costs in terms of units of the numeraire good. The inverse demand function  $D$  is linear with an intercept  $a$  and slope  $-(\delta + \gamma h)/L$ . As usual, optimal quantity per product is determined by the intersection of the marginal revenue  $MR$  and marginal cost  $c(\omega)$  curves. The difference is that the slope of the marginal revenue curve reflects both the own price effects and the brand-level price effect. The marginal cost curve includes the cost saving from process innovation. The optimized  $c(\omega)$  is downward-sloping as higher quantities make it more profitable to undertake process innovation.

Equation (2.3) and the inverse demand determine the optimal markup charged by the firm. Substituting for optimal quantity  $q = L(a - c(\omega))/2(\delta + \gamma h)$  and optimal process  $\omega = (cq/2r_\omega)^2$ , the perceived price elasticity is

$$\varepsilon = -(p/q)(dq/dp) = \frac{2}{1 - c/a} \left( 1 - \frac{c^2/4r_\omega}{(\delta + \gamma h)/L} \right).$$

As usual, the optimal markup ( $\mu \equiv p - c(\omega)$ ) is inversely proportional to the perceived price elasticity of demand implying  $\mu = p/\varepsilon$ . Markups and perceived elasticity reflect two key features. First, branded multiproduct firms face higher elasticities and choose lower markups due to canni-

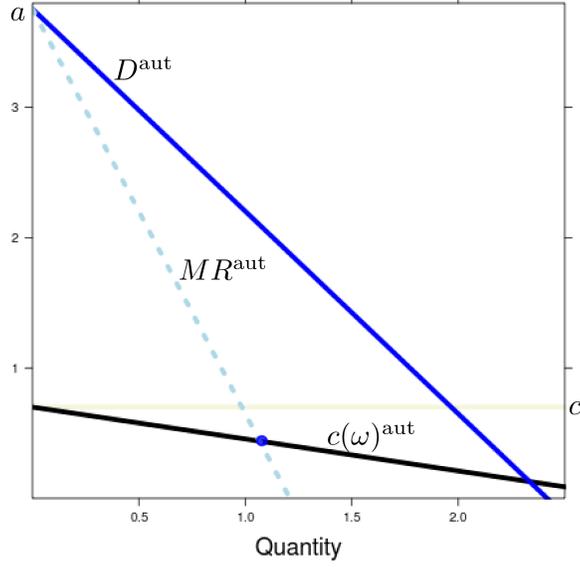


Figure 2.1: Optimal Quantity Choice

balization (through  $\gamma h$ ). When a firm introduces a new product, demand for its existing products falls. With linear demand for each of these products, this implies a rise in demand elasticity so multiproduct firms charge lower markups. Second, markups and elasticities respond to market demand conditions  $a$ . As market conditions deteriorate (i.e.  $a$  falls), the demand curve shifts inward implying a rise in demand elasticity. Firms perceive this rise in demand elasticity and respond by lowering markups. I will revisit this point when studying the impact of trade.

### 2.2.3 Product Range

At the optimal product range, profit from a new product  $\pi$  is equal to the fall in profit from cannibalization of old products. Adding a new product reduces the price of each old product by  $dp/dh = -\gamma q/L$ , resulting in total cannibalization of  $h(\gamma q/L)q$ . The FOC for product range  $h$  is

$$\frac{\partial \Pi}{\partial h} = \pi - \underbrace{h(\gamma q/L)q}_{\text{Cannib. Effect}} = 0. \quad (2.4)$$

Equation (2.4) ensures that the profit from the marginal product of a firm is driven down to zero. It shows that the net benefit from a new product falls with within-brand cannibalization  $\gamma$ , given other firm decisions ( $q$  and  $\omega$ ). Unlike process innovation, product innovation directly cannibalizes. Formally,  $\partial h(q, \omega, \gamma)/\partial \gamma < 0$  while  $\partial \omega(q, \gamma)/\partial \gamma = 0$ .

Product innovation can be interpreted as an instrument for firms to adjust demand elasticities. Equation (2.4) reflects a tradeoff between profits from a new product and higher elasticities of

all old products,  $\pi - h\pi'(\varepsilon)\partial\varepsilon/\partial h = 0$ . So product innovation enables firms to choose their optimal demand elasticities. A similar interpretation is given to “perceived quality” and horizontal differentiation in advertising and industrial organization (e.g. Dixit 1979, Rosenkranz 2003).

### 2.3 Equilibrium Outcomes in Autarky

Having determined firm decisions, I discuss equilibrium outcomes in autarky and show that cannibalization distinguishes product and process innovation in the market equilibrium.

From the firm FOCs, equilibrium quantity per product is  $q^{\text{aut}} = r_h^{1/2}/(\delta/L - c^2/4r_\omega)^{1/2}$ . As product R&D becomes more expensive (i.e.  $r_h$  rises), firms choose to increase quantities rather than products implying  $q$  and  $r_h$  are positively related. As process R&D becomes more expensive ( $r_\omega$  rises), firms find it less profitable to upgrade their production process implying  $q$  and  $r_\omega$  are negatively related. Optimal quantity  $q$  rises with market size  $L$  implying scale per product is higher in bigger markets.

Substituting for optimal quantity, process innovation is  $\omega^{\text{aut}} = [cq^{\text{aut}}/2r_\omega]^2 = (c/2r_\omega)^2 r_h/(\delta/L - c^2/4r_\omega)$ . Optimal process is independent of the degree of cannibalization  $\gamma$ , i.e.  $d\omega^{\text{aut}}/d\gamma = 0$ . Earlier, I showed that process innovation does not directly cannibalize (given  $q$ ). Now it can be seen that process innovation does not cannibalize even after taking other firm decisions into account ( $q^{\text{aut}}$  in this case).

Unlike process innovation, product innovation cannibalizes directly and indirectly (i.e.  $dh^{\text{aut}}/d\gamma < 0$ ). In equilibrium, free entry implies each firm earns zero total profit,  $\Pi = h\pi - f = 0$ . Substituting for profit from Equation (2.4) in the free entry condition, the product range in autarky is  $h^{\text{aut}} = [Lf(\delta/L - c^2/4r_\omega)/\gamma r_h]^{1/2}$ . Firms make fewer products when faced with higher within-brand cannibalization  $\gamma$ . I summarize these results in Proposition 1.

**Proposition 1.** *Product innovation cannibalizes directly and indirectly while process innovation does not. Formally,  $\partial h(q, \omega, \gamma)/\partial \gamma < 0$  and  $dh/d\gamma < 0$  while  $\partial \omega(q, \gamma)/\partial \gamma = d\omega/d\gamma = 0$ .*

Proposition 1 shows that an exogenous rise in the degree of within-brand cannibalization ( $\gamma$ ) does not alter process decisions of firms but lowers their product range. To understand this, it is useful to re-interpret firm decisions as choosing the optimal process  $\omega$ , quantity  $q$  and cannibalization  $b \equiv \gamma h q$ . If  $\gamma$  rises,  $b$  rises and the firm must re-optimize. Cannibalization does not directly affect  $\omega$  so a rise in  $\gamma$  leaves the optimal process choice unaffected; the process FOC is unchanged. The original quantity will also be optimal if the firm can lower  $h$  by an equivalent amount to keep  $b$  unchanged. With  $b$  unchanged, prices and profit lost from cannibalization are unaffected so the product range FOC holds with the new lower product range. Firms adjust to a rise in the degree of cannibalization  $\gamma$  through the extensive margin of products rather than the intensive margin of quantities (or processes). This is surprising, given that cannibalization depends on total brand-level

quantity  $hq$  and hence is symmetric in products and quantities. Intuitively, the reason why products cannibalize while quantities do not is that prices are more sensitive to cannibalization than marginal revenue. Product range is determined by price (through profit of the marginal product) while quantity is determined by marginal revenue of each product. Consequently, a rise in  $\gamma$  lowers returns to product innovation more than returns to quantities and process innovation. Firms therefore adjust to increases in  $\gamma$  through products and cannibalization does not play a role in process innovation.<sup>13</sup>

Equating the inverse demand function to the optimal price chosen by the firm, the mass of firms can be solved as  $M^{\text{aut}} = L[(\alpha - c)/q^{\text{aut}} + c^2/2r_\omega - 2(\delta + \gamma h^{\text{aut}})/L]/\eta h^{\text{aut}}$ . The mass of firms increases with  $L$  implying that bigger markets have more brands. This raises the mass of available products  $Mh$  showing that bigger markets have higher product variety, despite narrower brands.

## 2.4 Open Economy

I examine how trade affects innovation. Consider two identical countries, Home and Foreign. Following Melitz and Ottaviano (2008), suppose that home and foreign markets for differentiated goods are segmented. The homogeneous good is traded freely implying trade in differentiated goods need not be balanced. I discuss the impact of opening to trade on firm responses and welfare.

### 2.4.1 Equilibrium in an Open Economy

Opening the economy to free trade is equivalent to a rise in the size of an autarkic economy (from  $L$  to  $2L$ ). Firms sell the same quantities at home and abroad and total quantity per product rises after trade ( $q + q^x = 2q > q^{\text{aut}}$ ). This provides economies of scale and firms increase their process innovation to  $\omega^{\text{open}} = [c(2q)/2r_\omega]^2 = (c/2r_\omega)^2 r_h / (\delta/2L - c^2/4r_\omega) > \omega^{\text{aut}}$ . By symmetry of costs, firms sell the same product range at home and abroad and product innovation drops to  $h^{\text{open}} = [2Lf/\gamma(2q)^2]^{1/2} = [2Lf(\delta/2L - c^2/4r_\omega)/\gamma r_h]^{1/2} < h^{\text{aut}}$ . Opening to trade differentially affects the returns to product and process innovation. Firms engage in more process innovation at the expense of product innovation, as summarized in Proposition 2.

**Proposition 2.** *Opening the economy to trade reduces product innovation and increases process innovation within firms.*

The next sub-section explains the underlying economic reason and proceeds to a discussion of welfare gains and innovation policy.

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<sup>13</sup>This result is true for several variants of logit demand and nested CES demand (as shown in an online Appendix). Further, the reasoning is robust to oligopolistic competition and to a fixed mass of firms. Under oligopolistic competition, firms account for the change in market-level quantity when choosing their quantity and product range. However, prices continue to be more sensitive to cannibalization relative to marginal revenues and firms choose to make adjustments through product innovation rather than process innovation.

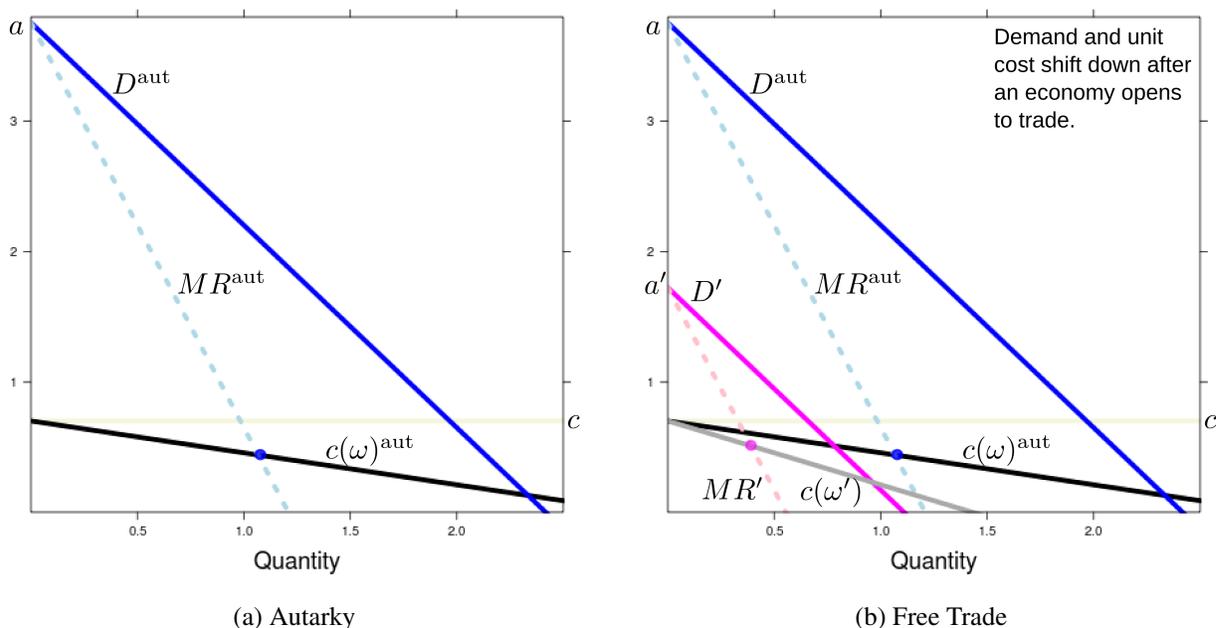


Figure 2.2: Direct Impact of Trade on Residual Home Demand

## 2.4.2 The Impact of Trade on Innovation

Trade increases the size of the home market which produces two effects: a market expansion effect and a product market competition effect. These two effects have opposing implications for firm innovation. I discuss each in turn.

The market expansion effect of trade on product and process innovation is straightforward. Trade provides firms with an opportunity to sell in the foreign market. This implies firms can increase the total quantity of each product as well as the marginal product. Consequently, access to the foreign market provides firms with an incentive to increase both product and process innovation. As shown in Figure 2.2 B, the marginal cost curve shifts down from  $c(\omega^{\text{aut}}) = c(\omega(q))$  to  $c(\omega') = c(\omega(q + q^x))$  after free trade. For ease of reference, the autarky figure is reproduced in Panel A and changes after trade are shown in Panel B.

Regarding the competition effect, we can trace out its implications through the demand function faced by a firm. When the home economy opens to trade, foreign firms anticipate higher profitability through exports and enter the home market. This lowers the demand intercept  $a \equiv \alpha - \eta Q/L$  as shown in Figure 2.2 B. Home demand for variety  $ij$  shifts down from  $D^{\text{aut}}$  to  $D'$ . The downward shift increases demand elasticities and lowers profitability. Firms counteract this rise in external competition by lowering internal competition through lower product innovation. Cutting product lines lowers the level of cannibalization. At the same time, it has a positive effect on prices implying profit per product increases. This fall in product innovation can also be interpreted in terms

of demand elasticities. Unlike the CES case, trade increases demand elasticities (through  $a$ ) and firms counteract the rise in elasticities by cutting product lines.<sup>14</sup>

### 2.4.3 The Role of Unbundling Innovation on Welfare

I discuss how incorporating product and process innovation affects welfare gains from trade. In standard models, trade increases welfare through variety and lower markups. I show innovation yields welfare gains which do not arise in the absence of a distinction between product and process innovation.

The indirect utility function of agent  $k$  is

$$V^k \equiv 1 + Mh(\alpha - p)^2 / 2(\delta + \gamma h + \eta Mh).$$

Standard welfare gains from trade arise through total variety  $Mh$  and price  $p$ . Consumers gain access to foreign brands and enjoy welfare gains from increase in product variety  $Mh$ . Although product innovation falls, new foreign varieties increase the total variety available to consumers. Rise in product variety lowers residual demand for each product and induces firms to decrease their markups  $\mu$ . This pro-competitive effect induces firms to lower prices ( $p = c(\omega) + \mu$ ) and leads to welfare gains from reduced markups.

Innovation affects welfare beyond these standard gains from variety and markups and, the distinction between product and process innovation is crucial in understanding these gains. Trade affects product innovation and increases access to varieties with lower demand elasticities. After trade, consumers enjoy access to more brands rather than more varieties from a few brands. Therefore, the product space features more differentiated varieties and welfare rises due to a fall in  $\gamma h$ . I refer to this rise in welfare from changes in firms' product range (given total variety  $Mh$ ) as welfare gains from product innovation. In models with no brand differentiation, there would of course be no welfare gains from product innovation. A less direct finding is that gains from product innovation would be absent if trade did not have differential effects on product and process innovation. As mentioned earlier, a nested CES demand model has no welfare gains from product innovation (despite brand differentiation) because trade does not alter the relative returns to innovation. In my

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<sup>14</sup>The reason for lowering product innovation can be understood from the response of the product FOC to a rise in market size  $L$ . The product FOC is  $\pi - \gamma h q^2 / L = 0$  implying product innovation is determined by change in profit from the marginal product relative to cannibalization, i.e.  $d\pi/dL - d(\gamma h q^2 / L)/dL = 0$ . Profit per product is  $\pi = [a - c(\omega) - \delta q/L - \gamma h q/L]q - r_\omega \omega - r_h$  implying  $d\pi/dL = q da/dL - (\gamma q^2/L)(dh/dL)$  from the envelope theorem. Rise in market size increases competition so  $da/dL = -(\delta + \gamma h)q/L^2 < 0$  from free entry. Substituting for  $da/dL$  and  $\gamma(hq)^2/L = f$  from free entry, the product FOC gives  $(\gamma q^2/L)(dh/dL) = q da/dL$ . Thus  $dh/dL < 0$  and cutting product lines enables firms to face tougher competition by lowering cannibalization. In models with CES demand, the negative impact of trade on product innovation arises due to increasing costs of product innovation or rising marginal cost of additional products. I have closed these channels so product adjustments work only through the interaction between competition and cannibalization (and not through cost linkages).

linear demand model, trade has differential effects on the returns to innovation and yields welfare gains.

As is well-understood, opening to trade provides scale economies and increases welfare through higher process innovation  $\omega$ . Firms attain cost savings ( $c(\omega)$  falls) and pass them to consumers, leading to welfare gains from lower prices. Unbundling innovation provides a new insight into how the interaction between product and process innovation affects welfare. If firms did not have the choice of process innovation (say  $r_\omega \rightarrow \infty$ ), it is immediate that welfare gains from cost savings would not arise. A less immediate result is that firms would not have an incentive to change product innovation either. In the absence of process innovation, opening to trade does not provide an incentive to shift innovation from product to process. There is no differential impact on the returns to innovation and firms continue with their original product range after trade. Process innovation provides firms with a more productive avenue to divert their investments. Therefore, considering the joint decision between product and process innovation introduces new welfare relationships which would not arise if the focus is only on total innovation. I summarize these results in Proposition 3.

**Proposition 3.** *Opening the economy to trade provides positive welfare gains from product innovation. In the absence of process innovation, there are no welfare gains from product innovation.*

#### 2.4.4 The Role of Unbundling Innovation on Policy

Distinguishing between product and process innovation has normative implications as well. As firms are imperfectly competitive, they need not internalize the benefits of innovation fully. I examine which type of innovation provides higher welfare gains and how the benefits from innovation change with trade liberalization.<sup>15</sup>

Let  $\tau_\omega$  denote subsidies to reduce the cost of process R&D from  $r_\omega$  to  $(1 - \tau_\omega)r_\omega$ . Similarly, let  $\tau_h$  denote subsidies for product R&D cost. To focus on the distinction between product and process innovation, I assume innovation policies are self-financed so that they only constitute a reallocation of resources from one form of innovation to another. Aggregate R&D support is zero  $Mh[\tau_\omega r_\omega \omega + \tau_h r_h] = 0$  so the product R&D subsidy is  $\tau_h = -\tau_\omega r_\omega \omega / r_h$ . To understand the benefits from supporting process innovation relative to product innovation, I examine how optimal innovation policies  $(\tau_\omega, \tau_h)$  change with trade.

Individual welfare can be written as  $U^k = 1 + Mhq(\alpha - p)/2L$ . Consequently, optimal  $\tau_\omega$  is given by  $d \ln Mhq / d \ln \tau_\omega + d \ln(\alpha - p) / d \ln \tau_\omega = 0$ . From the inverse demand function,  $\alpha - p = \delta q/L + \gamma hq/L + \eta Mhq/L$  so the effect of innovation policy on welfare consists of changes in variety-level quantity  $q$ , brand-level quantity  $hq$  and market-level quantity  $Mhq$ . Incorporating

<sup>15</sup>See Dhingra and Morrow (2012) for a general analysis of distortions under imperfect competition.

R&D support in the benchmark model, the optimal innovation policy is  $\tau_\omega = \eta Mh/2[\delta + \gamma h + 2\eta Mh]$  and  $\tau_h = -\tau_\omega r_\omega \omega / r_h$  (see Appendix). As  $\tau_\omega > 0 > \tau_h$ , process innovation is rewarded more than product innovation. Although this is a general equilibrium result, the underlying reason for higher consumer surplus from process innovation can be understood through the effects on firm decisions ( $q$ ,  $hq$ ) and market-level quantity ( $Mhq$ ).

A fall in product R&D costs induces firms to increase product variety  $h$  while a fall in process R&D costs induces firms to engage in process innovation and hence to increase quantities  $q$ . As products  $h$  and quantities  $q$  affect  $hq$  symmetrically, product and process R&D have the same effects on brand-level quantity. A drop in innovation costs through either process or product R&D subsidies induces firms to raise their brand-level quantity. Put differently, brand-level quantity depends on total innovation costs  $r_\omega \omega + r_h$  (and not separate R&D expenditure). Therefore, the effects of product and process R&D on brand-level quantity are the same. Similarly, firm entry depends on firm profitability and hence on total innovation costs. Both product and process R&D subsidies have the same effects on firm entry. The difference between returns to product and process innovation comes from their effects on variety-level quantity. Product R&D has no direct effect on quantity. On the other hand, process R&D increases quantity by lowering marginal costs. Quantity and process innovation reflect complementarities, and therefore, process innovation increases per unit consumer surplus more than product innovation.

Opening to trade increases the returns to process innovation relative to product innovation (because  $\tau_\omega$  rises as  $Mh$  rises). As explained earlier, differences in welfare from product and process R&D come from variety-level quantities. Opening to trade increases market size and reinforces the complementarity effect between variety-level quantity and process choice. As market size expands, scale increases and the relative returns to process innovation rise. Inadequate process innovation becomes more costly and there is a greater need to support process R&D. Unbundling innovation therefore shows how policymakers can target R&D support. The returns to supporting process R&D are higher after trade liberalization. Further, the relative returns to process innovation increase more in industries with brand differentiation. This is because higher entry of brands after trade lowers the returns to product innovation as consumers prefer to buy more varied brands. I summarize the key results in Proposition 4.

**Proposition 4.** *Optimal innovation policy supports process innovation relative to product innovation  $\tau_\omega > 0 > \tau_h$ . Opening to trade increases the benefits from supporting process innovation.*

Proposition 4 raises the issue of whether policymakers can be expected to have the information needed to target innovation policy. I argue this is a reasonable expectation as current forms of innovation support already require firms to furnish detailed information regarding their investment projects. For instance, under the US Research and Experimentation Tax Credit scheme, the Internal

Revenue Service requires firms to show that the eligible investment created a new or improved functionality, performance, reliability, or quality of a product or process.<sup>16</sup> The collection of such information is not limited to developed countries. A case in point is the flagship scheme of the Ministry of Textiles of India, the Technology Upgradation Fund Scheme (TUFS). TUFS requires firms to furnish details on the vintage of the machinery being purchased. It is noteworthy that the stated purpose of this scheme is to increase competitiveness of the textile industry to take advantage of market access provided by the Multi-Fiber Agreement.<sup>17</sup> As details of firm investment are routinely collected, policy insights from unbundling innovation can enable more effective targeting of innovation support.

### 3 Within-Brand and Across-Brand Competition

The previous Section provided a benchmark framework for the impact of trade liberalization on product and process innovation. The benchmark model assumed within-brand cannibalization is higher than across-brand competition. Broda and Weinstein (2010) find this is plausible for supermarket products. However, the assumption is unlikely to hold in industries such as electronics where product characteristics are at least as important as branding. This Section provides a richer demand specification and shows how within-brand and across-brand competition shape the impact of trade on innovation.

#### 3.1 Demand

Ideally, a general demand structure would allow for the possibility of different substitutability between any two varieties in the market. As is well-known, this poses severe dimensionality problems in both theoretical and empirical analyses (Zhelobodko et al. forthcoming, Nevo 2011). Consequently, I enrich the demand specification in two ways to provide results relevant for innovation and competition. First, I incorporate substitutability along the product dimension. Second, I consider the interaction between within-brand and across-brand competition.

The inverse demand of Section I is  $p_{ij} = \alpha - \delta q_{ij}/L - \gamma q_j/L - \eta Q/L$ . This implies variety  $i$  of brand  $j$  competes with all varieties in the market (through  $\eta Q$ ), but there is an asymmetry between competition among own varieties of a brand and varieties of other brands (due to  $\gamma q_j$ ). A more realistic approach is to think of  $i$  as a set of product characteristics and consider asymmetries along both the product dimension  $i$  and the brand dimension  $j$ . Then the inverse demand can be written as  $p_{ij} = \alpha - \delta q_{ij}/L - \gamma q_j/L - \eta Q_i/L$  where  $Q_i$  is the aggregate quantity of all products

<sup>16</sup>Source: Audit Techniques Guide: Credit for Increasing Research Activities (i.e. Research Tax Credit) IRC § 41 published in June, 2005. Available at <http://www.irs.gov/businesses/article/0,,id=156358,00.html>.

<sup>17</sup>Source: Ministry of Textiles, India. Available at [www.ministryoftextiles.gov.in/faq/faq\\_tuf.pdf](http://www.ministryoftextiles.gov.in/faq/faq_tuf.pdf)

that compete directly with the  $i$ th product characteristics. To fix ideas, suppose the  $i$ th variety of brand  $j$  only competes with the  $i$ th varieties of other brands. Then  $Q_i \equiv \int q_{ij'} dj'$  and variety  $ij$  is negatively affected by all varieties with the  $i$ th product characteristics produced across different firms  $j'$ . The inverse demand shows willingness to pay falls with higher brand-wide consumption ( $\gamma > 0$ ) and higher product-wide consumption ( $\eta > 0$ ). The across-brand price effect  $\eta$  denotes substitutability across varieties with similar product characteristics. If  $\gamma > \eta$ , then price is more sensitive to brand-level consumption rather than product-level consumption. Within-brand cross elasticity ( $|d \ln q_{ij} / d \ln q_{i'j}|$ ) is higher than across-brand cross elasticity ( $|d \ln q_{ij} / d \ln q_{ij'}|$ ) as in the benchmark model. On the other hand,  $\gamma < \eta$  implies product characteristics affect price more than branding and within-brand cross elasticity is lower than across-brand cross elasticity.

More generally, the set of product characteristics which affect variety  $ij$  can be broadened to  $Q_i = \int \omega_{ii'} (\int q_{i'j} dj') di'$ . This implies that variety  $ij$  competes with all  $i'$  product characteristics for which  $\omega_{ii'} > 0$ . The weight  $\omega_{ii'}$  captures the ease of substitutability between product characteristics  $i$  and  $i'$ . For example,  $\omega_{ii'} = 1_{i=i'}$  implies product-specific competition ( $Q_i = \int q_{ij'} dj'$ ) while  $\omega_{ii'} = 1$  implies variety  $ij$  competes directly with all varieties of all firms ( $Q_i = Q$ ). To capture both product-specific competition and competition across all product characteristics, we can set  $\omega_{ii'} = 1 + 1_{i=i'} \beta$  so that  $Q_i = Q + \beta \int_j q_{ij'} dj'$ . When  $\beta > 0$ , competition is stronger among varieties with the same product characteristics. Introducing weights  $\omega_{ii'}$  therefore provides flexibility by incorporating both brand and product dimensions in the inverse demand.

However, an implicit assumption of this inverse demand is that the brand and product dimensions are independent of each other. Higher consumption from a brand has no effect on consumers' willingness to pay for varieties from other brands (except through lower disposable income). To understand how within-brand competition affects across-brand competition, I consider the following inverse demand which allows for an interaction between these two effects:

$$p_{ij} = \alpha - \frac{\delta}{L} q_{ij} - \frac{\gamma}{L} q_j - \frac{\eta}{L} Q_i - \frac{\kappa}{L} q_j Q_i. \quad (3.1)$$

Inverse demand of Equation (3.1) introduces a brand-product effect ( $\kappa q_j Q_i$ ) in addition to the brand effect and the product effect. The importance of this new parameter  $\kappa$  is to capture the interaction between within-brand and across-brand competition. From Equation (3.1), the within-brand price effect is  $\partial p_{ij} / \partial q_j = -(\gamma + \kappa Q_i) / L < 0$  and the across-brand price effect is  $\partial p_{ij} / \partial Q_i = -(\eta + \kappa q_j) / L < 0$ . There is no restriction on the relative magnitudes of the within-brand and across-brand price effects and these can change endogenously (through changes in  $q_j$  and  $Q_i$ ). When  $\kappa > 0$ , higher brand-level consumption strengthens the negative price effect of across-brand consumption. This price effect is similar to within-brand cannibalization in that it lowers the inverse demand. A more interesting case is when  $\kappa < 0$  and the brand-product effect moves against

within-brand cannibalization. In this case, introduction of a new variety lowers the inverse demand due to within-brand cannibalization but exerts a positive influence on across-brand competition.

From the inverse demand of Equation (3.1), cross-elasticity of demand for  $ij$  with respect to another variety  $i'j'$  is  $\varepsilon_{ij,i'j'} = [\omega_{i'}(\eta + \kappa q_j) + 1_{j=j'}(\gamma + \kappa Q_i)](q_{i'j'}/\delta q_{ij})$ . In the benchmark model,  $\omega_{i'} = 1$  and  $\kappa = 0$  implying within-brand cross elasticity always exceeds across-brand cross elasticity. The introduction of  $\kappa \neq 0$  enriches the cross elasticity patterns in two ways. Cross elasticities can vary by brand and product characteristics. Further, the difference between within-brand and across-brand cross elasticities can change endogenously.

A natural question is how to interpret these richer brand-product demand effects in terms of welfare. I show that the brand-product demand effect captures consumer welfare from visibility under the following utility function:

$$U^k = q_0^k + \alpha Q^k - \frac{\delta}{2} \int_j \int_i (q_{ij}^k)^2 di dj - \frac{\gamma}{2} \int_j (q_j^k)^2 dj - \frac{\eta}{2} \int_i (Q_i^k)^2 di - \tilde{\kappa} \int_j \int_i q_j Q_i q_{ij}^k di dj.$$

The terms  $q_j$  and  $Q_i$  refer to aggregate brand-level and product-level consumption by all consumers in the market. When  $\tilde{\kappa} = \kappa/L$ , this utility function yields the inverse demand of Equation (3.1).<sup>18</sup> The new interaction term captures the effect of brand visibility and product visibility on consumer demand. Visibility can help consumers get access to a variety, but undermine its distinctiveness. For  $\kappa > 0$ , a high visibility variety is less desirable and consumers prefer varieties unique to the market. This is likely to be relevant in the textile and garments industry where consumers place a higher value on custom-made goods rather than mass-produced goods. For  $\kappa < 0$ , a high visibility variety is more desirable. We can think of  $\kappa < 0$  as summarizing lower consumer search costs and better reputation. Search costs are important in industries such as household goods (e.g. Gentry 2011) and reputation is more important in the electronics industry (e.g. Cabral 2000). The next sub-section examines the impact of trade on innovation under this richer demand structure.

## 3.2 Impact of Trade

I discuss how trade affects innovation under different demand characteristics. Firms face a product-specific intercept ( $\eta Q_i/L$ ) and account for the interaction of within-brand and across-brand competition ( $\kappa q_j Q_i/L$ ). This introduces a product location choice as firms must decide on the set of products they produce. In equilibrium, all products must be equally profitable as a firm would otherwise relocate to a more profitable product. Therefore,  $Q_i$  is the same for all products and the firm problem is similar to Section I.

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<sup>18</sup>Specifying  $\tilde{\kappa}$  as  $\kappa/L$  makes the algebra in the next sub-sections less tedious as it avoids inclusion of home exports in the utility function of home consumers.

Firms make their process, quantity and product decisions, and free entry determines the mass of entrants. From the firm's FOC for process choice, optimal production process is determined by  $-c'(\omega)q = r_\omega$  in autarky. Optimal quantity is given by the equality of marginal revenue to marginal cost,  $p - (\delta + \gamma h + \kappa h Q_i)q/L = c(\omega)$ . Firms take the impact of quantity on across-brand competition into account. When  $\kappa > 0$ , they can charge higher markups due to the uniqueness of their variety in the market. The FOC for product range is  $\pi - h[(\gamma + \kappa Q_i)q/L]q = 0$  which shows that firms account for the price effects through both cannibalization and across-brand competition. Free entry implies  $h\pi = (\gamma + \kappa Q_i)(hq)^2/L = f$ .

As in Section I, equilibrium quantity is  $q^{\text{aut}} = r_h^{1/2} / (\delta/L - c^2/4r_\omega)^{1/2}$  and  $\omega^{\text{aut}} = (cq^{\text{aut}}/2r_\omega)^2$  implying process innovation is independent of cannibalization. Product range is  $h^{\text{aut}} = (fL)^{1/2} / (\gamma + \kappa Q_i^{\text{aut}})^{1/2} q^{\text{aut}}$  where  $Q_i^{\text{aut}} = (L/\eta) \left( \alpha - c - 2r_h/q^{\text{aut}} - 2f/(hq)^{\text{aut}} \right)$ . Firms offer fewer products when faced with a higher rate of cannibalization ( $dh/d\gamma = -\eta h/2(\gamma + \kappa Q_i)(\eta + \kappa hq) < 0$ ) or a higher rate of across-brand competition ( $dh/d\kappa = -\eta h Q_i/2(\gamma + \kappa Q_i)(\eta + \kappa hq) < 0$ ).

Opening the economy to free trade is equivalent to a rise in the size of an autarkic economy (from  $L$  to  $2L$ ). As earlier, total quantity per product rises and firms are induced to increase their process innovation. However, product innovation now depends on the interplay between the change in cannibalization and perceived across-firm competition. As market size increases, firms change the product range as follows:

$$d \ln h / d \ln L = -\frac{L}{2} \left[ \frac{\kappa h q}{\eta + \kappa h q} \frac{\delta + \gamma h + \kappa h Q_i}{\gamma h + \kappa h Q_i} + \frac{c^2/4r_\omega}{\delta/L - c^2/4r_\omega} \right]. \quad (3.2)$$

When  $\kappa = 0$ , we know from Section I that firms reduce their product range ( $d \ln h / d \ln L < 0$ ). When consumers value uniqueness of a variety ( $\kappa > 0$ ), firms perceive the negative effect of product innovation on across-brand competition and lower product innovation further. In fact, the higher the rate of across-brand price effect, the higher is the drop in product innovation after trade ( $|d \ln h / d \ln L|$  is increasing in  $\kappa$ ). When consumers value visibility of a variety ( $\kappa < 0$ ), firms internalize the positive effect of product range on reducing across-brand competition. This positive across-brand price effect can overcome the negative price effect from higher cannibalization, and induce firms to increase product innovation. The higher the degree of visibility, the higher is the positive across-brand price effect and rise in product innovation. Let  $\hat{\kappa}$  be the cutoff value at which  $d \ln h / d \ln L = 0$ . Then product innovation rises for  $\kappa < \hat{\kappa}$  and falls for  $\kappa > \hat{\kappa}$ . I summarize the results for the impact of trade on innovation in Proposition 5.

**Proposition 5.** *Opening the economy to trade increases process innovation. Product innovation falls in industries with high brand-product demand effect ( $\kappa > \hat{\kappa}$ ), but rises in industries with low brand-product demand effect ( $\kappa < \hat{\kappa}$ ).*

Proposition 5 shows the impact of trade on product innovation depends on demand characteristics and is therefore likely to vary across markets. Empirical work points to these market-specific differences. Gorodnichenko et al. (2010) find a positive relationship between higher foreign competition and product and process innovation in transition countries. But Baldwin and Gu (2004, 2006) find Canadian firms reduced product innovation and increased process innovation after the Canada-US free trade agreement. Under high brand-product demand effects, firm innovation responses are qualitatively similar to the benchmark model where  $\kappa = 0$ . Therefore, I focus on the case of low brand-product effects and contrast the welfare results with those for the benchmark model.

### 3.3 Welfare

To derive welfare results, I need to specify the product dimension in the inverse demand. For flexibility, suppose product  $i$  competes directly with all products but more strongly with  $\mathcal{J}_{s_i}$  similar products in the market. Specifically, let the weights on the product dimension be  $\varpi_{i'} = 1 + 1_{i' \in \mathcal{J}_{s_i}} \beta$ . Then  $Q_i = Q + \beta \int_{\mathcal{J}_{s_i}} (\int q_{i'j'} d j') d i'$  and we can interpret  $s_i \equiv \int_{\mathcal{J}_{s_i}} (\int q_{i'j'} d j') d i' / Q$  as the fraction of products with characteristics similar to  $i$ . Let  $s \equiv 1 + \beta s_i$ , then the indirect utility function of agent  $k$  is

$$V^k \equiv 1 + \frac{1}{2} Mh(\alpha - p)^2 \left( \frac{\delta + \gamma h + \eta s Mh}{(\delta + \gamma h + \kappa h Q_i + \eta s Mh)^2} \right) \quad (3.3)$$

where  $Q_i$  is implicitly defined as  $Q_i/L = s Mh(\alpha - p) / (\delta + \gamma h + \kappa h Q_i + \eta s Mh)$ .

As in the benchmark model ( $\kappa = 0$ ), opening the economy to trade provides welfare gains from rise in total variety ( $Mh$ ). Consumers enjoy lower prices after trade because process innovation rises and markups fall (due to higher  $Q_i$ ). Although brand differentiation induces firms to lower product innovation, brand-product demand effects have a positive effect on product innovation as firms internalize lower across-brand competition from visibility. Opening to trade increases product innovation and provides welfare gains beyond standard gains from greater variety. As earlier, this gain from product innovation would not arise in the absence of a distinction between product and process innovation (when  $\gamma + \kappa Q_i = 0$ ). Opening to trade changes the difference between within-brand and across-brand cross elasticities. Within-brand cross elasticity falls due to higher  $Q_i$  and product innovation therefore increases access to low elasticity varieties. From Equation (3.3), the term in parenthesis is increasing in product innovation and trade provides welfare gains from higher product innovation.

### 3.4 Policy

The effect of trade liberalization on the relative benefits from product and process innovation are robust to different demand characteristics. However, considering brand-product demand effects provides further insights into optimal policies. Trade liberalization affects both within-brand and across-brand competition, and its effects on product innovation relative to firm entry depend on demand characteristics.

To understand these effects, I examine policies to subsidize the costs of process innovation  $\tau_\omega$ , product innovation  $\tau_h$  and firm entry  $\tau_e$ . Let  $RB_{\omega h}$  denote the relative benefit-to-cost ratio of supporting process innovation versus product innovation. Then  $RB_{\omega h} = (dU/d\tau_\omega)/Mhr_\omega\omega - (dU/d\tau_h)/Mhr_h$ . Differentiating welfare with respect to policy, it is straightforward to show  $RB_{\omega h} = \left(-c'/c''\right)(\delta q/L)/\left(2\delta q/L - (c')^2/c''\right)$ . As in the benchmark model, R&D support for process innovation provides higher benefits than product innovation ( $RB_{\omega h} > 0$ ). These benefits rise with market size, implying trade liberalization makes inadequate process innovation more costly.

A new insight arises when we account for the relation between within-brand and across-brand competition. The benefit from supporting firm entry relative to product innovation is  $RB_{eh} = (\delta q/L)/\left(2\delta q/L - (c')^2/c''\right) - \gamma/2(\gamma + \kappa Q_i)$ . In the absence of brand-product demand effects, encouraging entry of brands provides higher welfare than product innovation as consumers value brand differentiation. Opening to trade increases the need to support entry relative to product innovation. This is because product subsidies induce firms to increase product innovation at the expense of variety-level quantity. As the importance of economies of scale rises after trade, the returns to product innovation fall further. With brand-product demand effects  $\kappa < 0$ , the need to encourage entry over product innovation is lower in industries with high visibility. This is intuitive as consumers value large brands in these industries. Trade liberalization further reduces the benefits of encouraging entry of brands relative to product innovation. Product visibility increases and the benefits from product innovation rise. Trade therefore reduces the need to subsidize entry of brands.

## 4 Firm Heterogeneity and Innovation

Sections I and II show how opening to trade affects average product and process innovation. As is well-known, there is substantial firm heterogeneity within industries. Recent empirical work finds innovation responses vary systematically across firms. This Section examines the impact of trade on product and process innovation across firms that differ in initial productivity. I start with a brief exposition of firm choices and then study the impact of different types of trade liberalization on

innovation.

## 4.1 Firm Heterogeneity under Restricted Trade

As earlier, firms pay an entry cost  $f$  to produce a brand of products with cost draw  $c$ . The cost draw is no longer deterministic. Firms know the distribution of costs  $c \sim G(c)$  defined on the support  $[0, c_M]$ . They do not observe the realizations before paying entry costs. Having paid the entry cost, each firm observes its cost draw  $c$ .<sup>19</sup> It decides whether to stay in the market or to exit immediately. No new information is revealed after the decision to stay. If a firm stays, it decides on its process, quantities and product range. In this sub-section, I consider discrete changes in technology for tractability and empirical conformity. The only difference from the homogeneous firm case is that the process decision does not involve choosing the level of upgrading. This simplifies the analysis without sacrificing richness in model predictions. By paying  $r_\omega$ , a firm with initial cost draw  $c$  can upgrade its process to  $c - \omega(c)$  where  $\omega'(c) \leq 0$ .<sup>20</sup>

To provide testable predictions, I consider the empirically relevant cases of unilateral and bilateral reductions in trade costs. A foreign tariff  $t^*$  increases the unit cost of exporting from  $c(\omega)$  to  $c(\omega) + t^*$  for home producers of the differentiated goods. A home tariff  $t$  increases the unit cost of exporting from  $c(\omega)$  to  $c(\omega) + t$  for foreign firms.

## 4.2 Firm Choices and Equilibrium

With firm heterogeneity, firm choices are determined in a manner similar to Sections I and II. I relegate details to the Appendix and provide a brief exposition of firm choices under costly trade.

A home firm chooses production processes, quantities and product range to maximize

$$\max_{\omega, q, q^x, h} \Pi(c) = h[(p^d - c + 1_{\omega>0}\omega(c))q^d + (p^x - c + 1_{\omega>0}\omega(c) - t^*)q^x - 1_{\omega>0}r_\omega - r_h]$$

where  $*$  denotes Foreign, and  $1_{\omega>0} = 1$  if a firm invests in process innovation and 0 otherwise. Due to the presence of trade costs, a key difference in market outcomes is that firms sell different quantities in the domestic and export markets. Optimal quantity sold domestically is  $q^d(c) = L(a - c(\omega))/2(\delta + \gamma h + \kappa h Q_i)$  and optimal export quantity is  $q^x(c) = L(a^* - t^* - c(\omega))/2(\delta + \gamma h + \kappa h Q_i^*)$  where  $a \equiv \alpha - \eta Q_i/L$ , and  $a^* \equiv \alpha - \eta Q_i^*/L$ . The export to domestic sales ratio of a home firm with cost draw  $c$  is  $\theta(c) \equiv q^x(c)/q^d(c)$ . This ratio captures the firm-specific increase in market size from exporting. Specifically, exporting increases the market size available to firm  $c$

<sup>19</sup>I abstract from within-firm heterogeneity and rising costs of product innovation to simplify the analysis and show that changes in product range are not driven by cost linkages.

<sup>20</sup>I assume returns to process innovation are increasing in initial productivity as in the empirical findings of Bustos (2009) and Bas (2008).

from  $L$  to  $z(c)L$  where the size factor is  $z(c) \equiv (1 + \theta(c))^2 / (1 + \theta(c)^2)$ . As  $\theta(c)$  rises, size factor  $z(c)$  rises implying firm  $c$  experiences an increase in available market size. The rise in firm size can be seen from the total quantity choice,  $q^d(c) + q^x(c) = z(c)^{1/2}(r_h + 1_{\omega>0}r_\omega)^{1/2}(L/\delta)^{1/2}$ , which follows from the quantity and product FOCs of the firm.

With the quantities in hand, process choice can be determined by comparing profits with and without upgrading. Using  $\theta(c)$ , total profits of a firm are:

$$\Pi = \frac{L(1 + \theta^2 \lambda^x / \lambda)}{4(\gamma + \kappa Q_i)} \left[ a - c(\omega) - 2(\delta/L)^{1/2} (1_{\omega>0}r_\omega + r_h)^{1/2} / (1 + \theta^2)^{1/2} \right]^2$$

where  $\lambda \equiv (\delta + \gamma h + \kappa h Q_i)^2 / (\gamma + \kappa Q_i)$  and similarly for  $\lambda^x$ . Let  $c_{0\omega}$  refer to a firm that is indifferent between not upgrading and upgrading its process so that  $\Pi(c_{0\omega}) = \Pi(c_{0\omega} - \omega(c_{0\omega}))$ . To understand the determinants of process innovation, suppose  $c_{0\omega}$  is a non-exporter. Then process innovation is viable as long as  $\omega(c) \geq 2(\delta/L)^{1/2} \left[ (r_\omega + r_h)^{1/2} - r_h^{1/2} \right]$ . As  $\omega'(c) \leq 0$ , all non-exporters with cost draws lower than  $c_{0\omega}$  engage in process innovation while higher cost non-exporters do not. For exporters, the logic is similar though the expressions are more cumbersome as  $\theta$  depends on  $\omega$  as well (see Appendix).

The process innovation rule and the three conditions for  $q^d(c)$ ,  $q^x(c)$  and  $q^d(c) + q^x(c)$  determine firms' optimal quantities, process and product range in terms of aggregate quantities ( $Q_i$  and  $Q_i^*$ ). Aggregate quantities in turn depend on free entry of firms in the home and foreign countries. This pins down all equilibrium outcomes, and we can proceed to studying the impact of trade liberalization on innovation.

### 4.3 The Impact of Trade Liberalization on Innovation

This sub-section discusses the effects of a bilateral tariff liberalization and compares it with a unilateral reduction in home tariffs. I then place the results in the context of previous work on innovation.

#### 4.3.1 Bilateral Trade Liberalization

A bilateral trade liberalization reduces tariffs in both countries and increases the export to domestic production ratios of home and foreign exporters. But now the rise in market size ( $z(c) = (1 + \theta(c))^2 / (1 + \theta(c)^2)$ ) differs across firms. I discuss the effects on non-exporters and exporters in turn. Continuing non-exporters do not experience any change in market size as their exports continue to be zero. Their export to domestic production ratio  $\theta$  and hence their size factor  $z$  is unaffected after trade. Consequently, non-exporters do not change their process decisions. This can be seen directly as  $c_{0\omega}$  is independent of trade costs for non-exporters. However, non-exporters

change their product choice as they are adversely affected by tougher import competition. I discuss this in detail.

Trade liberalization induces entry of foreign firms and toughens competition (by increasing  $Q_i$ ). This reduces the residual demand by lowering the demand intercept  $a \equiv \alpha - \eta Q_i/L$ . Continuing non-exporters experience no rise in market size and instead are faced with tougher competition in the home market. Cannibalization induces them to lower competition by cutting back on product lines. The full force of tougher competition is realized in the form of lower product range rather than lower quantity per product. This is not due to a discrete process choice, rather due to the differential effects of trade on the returns to innovation. The reasoning is similar to that for exogenous changes in cannibalization. Price is more sensitive to market demand conditions implying that returns to product innovation decline more than returns to quantity expansion. As a result, non-exporters adjust to tougher competition by narrowing their product range.

New and continuing exporters experience an expansion in market size ( $z(c)$ ) after trade liberalization. Access to foreign markets makes it easier to engage in process innovation ( $dc_{0\omega}/dt < 0$ ). At the same time, they face tougher competition. The relative strength of the market expansion and competition effects determines product innovation. The lowest cost exporters are able to capture higher market shares which provides them with higher scale economies and visibility. This enables them to absorb higher within-brand cannibalization from more product lines. Higher cost exporters supply predominantly to the home market where competition has become more intense. Market expansion through trade is not enough to undo their loss from worse home market conditions. Consequently, high cost exporters cut back on product lines to counteract the rise in demand elasticities. Formally,  $dh/dt$  is positive for non-exporters but decreases with  $c$  and therefore negative beyond a cost cutoff. When  $\kappa = 0$ , the cost cutoff for higher product innovation is precisely the cost of the average exporter (i.e.  $\tilde{c}$  such that  $\theta(\tilde{c}) = \int hq^x dG / \int hq^d dG$ ). All firms with cost draws below  $\tilde{c}$  increase product innovation while firms with cost draws above  $\tilde{c}$  cut back on product lines. I contrast the innovation responses from a bilateral liberalization with a unilateral home tariff reduction in the next sub-section.

### 4.3.2 Unilateral Trade Liberalization at Home

A unilateral home tariff cut reduces the tariff faced by foreign exporters. The direct impact is a rise in foreign firms' export shares  $\theta^*$  (given  $a$  and  $a^*$ ). With a rise in  $\theta^*$ , foreign firms expect a rise in market size available to them. This implies more entry and tougher competition in the foreign market, leading to a deterioration in market-wide demand conditions  $a^*$ . In the home market, firms expect a fall in market size available to them due to higher imports at home. This induces exit in the home economy. Exit of home firms improves market-wide demand conditions  $a$  at home. The indirect impact of a fall in  $a^*$  and a rise in  $a$  reinforces the rise in market size through  $\theta^*$ . Foreign

exporters experience a rise in market size at the expense of home firms. I discuss the impact of these changes on home firms.

Non-exporting home firms face tougher competition but experience no market expansion (because  $\theta$  and hence  $z$  continue to be zero). As earlier, they reduce their product lines and do not change their process choice. Compared to a bilateral trade liberalization, a unilateral home tariff cut lowers market size for home exporters and induces the opposite innovation responses. Home exporters lose market size to foreign firms and are less likely to engage in process innovation. This is due to the indirect impact of a fall in  $a^*$  and a rise in  $a$  on their size factor. High cost exporters sell mainly at home and benefit from exit of home firms. They are able to increase their product lines. Low cost home exporters face the toughest competition as they supply mostly to the foreign market and engage in lower product innovation. For ease of reference, I summarize the impact of trade liberalization on innovation in Proposition 6.

**Proposition 6.** *With a bilateral tariff reduction or a unilateral foreign tariff reduction, exporters are more likely to engage in process innovation. Low-productivity exporters and non-exporters reduce product innovation while high-productivity exporters engage in higher product innovation. A home tariff reduction has the opposite effects.*

Proposition 6 shows that product and process innovation move in the same direction for large exporters but in opposite directions for non-exporters. Thus product and process innovation reflect complementarities for large exporters (as in Athey and Schmutzler 1995) but substitutability for non-exporters (as in Eswaran and Gallini 1996). This heterogeneity in firm responses is consistent with differences in product innovation among Canadian firms during CUSFTA. Baldwin and Gu (2006) find small Canadian firms lowered product innovation while large Canadian exporters increased product innovation during CUSFTA. Notably, Lileeva and Trefler (2010) find productivity gains of Canadian plants are positively related to output *per product*, suggesting a role for scale economies at the product level. Within Argentinean manufacturing, Bustos (2009) finds the expected result that foreign tariff cuts induce exporters to engage in greater product and process innovation, relative to non-exporters.<sup>21</sup> Similarly, Iacovone and Javorcik (2010) show that existing Mexican exporters increased product variety by more than new exporters after the US tariff cuts of NAFTA. They also find higher investments in physical capital among exporters, relative to non-exporters. These studies consider bilateral trade liberalization but a related literature on plant size provides supporting evidence for unilateral trade liberalization. In my model, a home tariff cut

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<sup>21</sup>The product innovation measure of Bustos includes both new products and “technological improvement of existing products.” Similarly, Teshima (2008) includes product quality upgrades in his product R&D measure of Mexican firms. He shows Mexican tariffs are positively correlated with process R&D but statistically uncorrelated with product R&D in 2000-2003. Unfortunately, it is difficult to interpret his findings in the context of my model for two reasons. First, quality upgrades are not isomorphic to introduction of new products in my model. Second, Mexican tariff changes may be correlated with US tariff changes during the period.

induces small exporters and non-exporters to engage in product innovation and expand plant size  $h(q^d + q^x)$ . Indeed, Tybout et al. (1991) find that lower protection enabled small Chilean plants to expand output and the plant size distribution became more uniform in industries experiencing large home tariff cuts. Though not unique to this paper, the contrasting effects of foreign and home tariff cuts are noteworthy. Head and Ries (1999) find the size of Canadian establishments declined due to Canadian tariff cuts but increased due to US tariff cuts, a finding which is consistent with my model.

The results of Proposition 6 can also be compared with recent work on multiproduct firms. The literature on multiproduct firms explains differences in products sold to different markets so the focus is on within-firm heterogeneity and selection of better products as the driving force for observed increases in productivity. Though my question is different, the model has implications for productivity and products. As in Bernard et al. (2011) and Mayer et al. (2009), productivity of exporters increases after a bilateral trade liberalization. The rise in productivity in my model comes from selection of higher productivity firms and from process innovation among existing firms. Following Mayer et al., the relative contribution of the selection and innovation channels to aggregate productivity growth can be expressed in terms of output per worker  $\bar{\Phi} = Q/M_e \int_0^{c^M} c(\omega) h q dG$ . Let  $c_d$  denote the cost cutoff for a firm that is indifferent between producing and exiting. Then brand-level output of firm  $c$  is  $h q = [c_d - c + 1_{c \leq c_{0\omega}} (\omega(c) - \omega(c_{0\omega}))] L/2 (\gamma + \kappa Q_i)$  and average output per worker in the economy is

$$\bar{\Phi} = \frac{\int_0^{c_d} (c_d - c) dG + \int_0^{c_{0\omega}} (\omega(c) - \omega(c_{0\omega})) dG}{\int_0^{c_d} (c_d - c) (c - 1_{c \leq c_{0\omega}} \omega(c)) dG + \int_0^{c_{0\omega}} (\omega(c) - \omega(c_{0\omega})) (c - \omega(c)) dG}$$

Output per worker rises due to selection (as  $c_d$  falls) and a rise in process innovation (as  $c_{0\omega}$  rises). Details of changes in firm selection and innovation are illustrated in the Appendix. A key difference in my model is that productivity growth of existing firms is driven by process innovation from higher scale (and not from product selection). Therefore, productivity of exporters rises even when they increase product variety but productivity of continuing non-exporters is unaffected. Bernard et al. (2011) and Mayer et al. find instead that firm productivity rises when product variety falls and that all firms show higher revenue-based productivity as they drop their marginal products. I find that decisions to drop products vary by firm productivity and export orientation.

## 5 Conclusion

Firms face competing needs to invest in product and process innovation. This paper introduces a framework to study the impact of market forces on these investments. In this framework, within-brand cannibalization distinguishes product and process innovation. A firm's new product canni-

balizes its old products while a new process has no cannibalization effects. This has consequences for the impact of competition on innovation strategies of firms. Focusing on trade policy, I provide new results for the impact of trade on market forces and firm innovation.

Opening to trade provides an opportunity to supply to a larger market. At the same time, trade makes competition fiercer and firms are faced with higher demand elasticities. These two forces of market expansion and tougher competition shape firm innovation. Market expansion results in greater process innovation through economies of scale. Tougher competition induces firms to lower cannibalization by reducing product lines. Having fewer products reduces visibility of a brand and exposes it to higher across-brand competition. The relative strength of within-brand cannibalization and across-brand competition determines the impact of trade liberalization on product innovation. Markets with high visibility have higher product innovation and this reduces the need to encourage entry after trade. The benefits from process innovation rise after trade as market expansion strengthens the complementarity between scale and process innovation. At the individual level, the impact of trade liberalization on innovation differs by firm productivity and export orientation. Large exporters get a sufficient boost in market size and visibility to outweigh the deterioration in market conditions at home. They engage in greater product innovation. Process innovation remains unaffected among non-exporters but increases among exporters as they expand output per product to supply to the foreign market.

These findings reveal how trade policy affects innovation. Tybout and Westbrook (1995) propose that the bulk of production gains accrue within firms through factors such as process rationalization, product expansion, capacity utilization and changes in lengths of production runs. They conclude that “much remains to be done in documenting the relative importance of these effects.” Empirical work has begun to address the role of trade in influencing these factors. Building on these insights, I characterize how firms innovate and how their responses shape production gains and innovation policy. The theoretical and empirical issues involved in unbundling innovation are formidable. This paper conceptualizes some of the issues but several questions merit further investigation. Fortunately, new plant-level surveys on innovation activities are increasingly becoming available, and future work can provide more insight into dynamics, international spillovers and policies fostering innovation.<sup>22</sup>

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<sup>22</sup>See Costantini and Melitz (2008) and Ederington and McCalman (2008) for dynamic aspects and Eckel et al. (2009) for cross-industry differences.

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## A Appendix

### A.1 Assumptions

Let  $\alpha > c + 2(\gamma f/L)^{1/2}, 2\eta^{1/2}$  to ensure consumption of both homogeneous and differentiated goods in equilibrium, as in Melitz and Ottaviano (2008). For well-defined profits and pro-competitive effects,  $\gamma + \kappa Q_i > 0$  and  $\eta + 2\kappa hq > 0$ .<sup>23</sup> I assume  $\delta/L > c'(\omega)^2[a - c(\omega)]/2c''(\omega)[r_\omega\omega + r_h]$  for all  $\omega > 0$  to ensure a strictly concave firm problem. This guarantees that quantity and process choices are such that the relative rate of decline in own marginal revenue from higher quantity is greater than the rate of decline in cost savings with a better process. For this purpose, I assume  $\delta/L > c'(\omega)^2[a - c(\omega)]/2c''(\omega)[r_\omega\omega + r_h]$  for all  $\omega > 0$ . For  $c(\omega) = c - c\omega^{1/2}$ , a sufficient condition in terms of primitives is

$$\delta/L > c \left[ r_h^{1/2}(\delta/L - c^2/4r_\omega)^{1/2} + (\gamma f/L)^{1/2} + c/2 \right] / 2r_\omega^{1/2}r_h^{1/2}$$

Positive unit costs after innovation are ensured by  $\delta/L > c^2(r_\omega + r_h)/4r_\omega^2$  which is consistent with the above concavity condition.

### A.2 R&D Support

To determine the optimal  $\tau_\omega$ , I solve for changes in total purchases  $d \ln Mhq/d \ln \tau_\omega$  and per unit consumer surplus  $d \ln(\alpha - p)/d \ln \tau_\omega$ . Substituting for  $\tau_h = -\tau_\omega r_\omega \omega / r_h$ , optimal quantity is  $\delta q^2/L = (1 - \tau_\omega)r_\omega\omega + (1 - \tau_h)r_h = r_\omega\omega + r_h$  and optimal process choice is  $-c'(\omega)q = (1 - \tau_\omega)r_\omega$ . From these choices,  $dq/d\tau_\omega = -c'(\omega)r_\omega / [2(\delta/L)qc''(\omega)(1 - \tau_\omega) - c'(\omega)^2]$  and  $d\omega/d\tau_\omega = 2(\delta q/r_\omega L)(dq/d\tau_\omega)$ . As expected, a higher reward to process innovation increases variety-level quantity and process innovation (because firm SOCs ensure that the term in square brackets is positive). Brand-level quantity  $hq$  determines firm size and hence firm profitability. From the product FOC ( $\pi = \gamma hq^2/L$ ) and free entry ( $h\pi = \gamma(hq)^2/L = f$ ), firm profitability and hence brand-level quantity are not affected by innovation policy  $dhq/d\tau_\omega = 0$ . However, market-level quantity is affected by innovation policy through firm entry. From the firm’s pricing FOC ( $p - c(\omega) = (\delta + \gamma h)q/L$ ), the change in market-level quantity is  $(\eta/L)(dMhq/d\tau_\omega) = -2\delta(dq/d\tau_\omega)/L - c'(\omega)d\omega/d\tau_\omega$ . Substituting for change in process,  $(\eta Mhq/L)(d \ln Mhq/d \ln \tau_\omega) =$

<sup>23</sup>A sufficient condition is  $\kappa \geq -\eta\gamma/4(\alpha - c)L$ .

$-2\tau_\omega(\delta q/L)(dq/d\tau_\omega)$ . Substituting for changes in  $q$ ,  $hq$  and  $Mhq$  shows change in per unit consumer surplus is  $d\ln(\alpha - p)/d\ln\tau_\omega = -(2\tau_\omega - 1)[\delta/(\delta + \gamma h + \eta Mh)](d\ln q/d\ln\tau_\omega)$ . Support for process R&D increases per unit consumer surplus (starting from  $\tau_\omega = 0$ ). From changes in per unit consumer surplus and market-level quantity, the optimal innovation policy is  $\tau_\omega = \eta Mh/2[\delta + \gamma h + 2\eta Mh]$  and  $\tau_h = -\tau_\omega r_\omega \omega / r_h$ .

### A.3 Heterogeneous Firms

Following Nocke and Yeaple (2005), I consider tariff changes evaluated in an interior equilibrium starting from  $t = t^* > 0$ . I first derive the changes in aggregate conditions and then discuss firm responses.

#### A.3.1 Impact on Aggregate Demand

The free entry conditions determine changes in aggregate demand. Assuming  $G(c)$  is such that the set of producers is convex, free entry of home firms implies  $\int_0^{c^M} (\partial\Pi(c)/\partial t^*) dG = 0$ . From the envelope theorem,  $\partial\Pi(c)/\partial t^* = hq^d (\partial p^d/\partial t^*) + hq^x (\partial p^x/\partial t^*) - hq^x$  where the change in prices is  $\partial p^d/\partial t^* = -(\eta + \kappa hq^d)(\partial Q_i/\partial t^*)/L$  and  $\partial p^x/\partial t^* = -(\eta + \kappa hq^x)(\partial Q_i^*/\partial t^*)/L$ . Substituting for price changes, free entry of home firms implies  $(dQ_i/dt^*) \int hq^d (\eta + \kappa hq^d) dG + (dQ_i^*/dt^*) \int hq^x (\eta + \kappa hq^x) dG = -\int hq^x (\eta + \kappa hq^x) dG$ . Free entry shows changes in aggregate demand are  $(A^x / \int hq^x dG) (dQ_i/dt^*) = -(A^x/A^d) / (1 - (A^x/A^d)^2)$  and  $(A^x / \int hq^x dG) (dQ_i^*/dt^*) = (A^x/A^d)^2 / (1 - (A^x/A^d)^2)$  where the aggregate terms are  $A^x \equiv \int hq^x (\eta + \kappa hq^x) dG$  and  $A^d \equiv \int hq^d (\eta + \kappa hq^d) dG$ . For a bilateral liberalization, changes in home and foreign aggregate demand are the same, and are given by  $(A^x / \int hq^x dG) (dQ_i/dt^*) = -(A^x/A^d) / (1 + (A^x/A^d))$ . As  $\eta + 2\kappa hq^x > 0$ ,  $A^x/A^d$  lies between 0 and 1. For  $\kappa = 0$ ,  $A^x/A^d$  is simply the average export to domestic sales ratio in the economy  $A^x/A^d = \tilde{\theta} \equiv \int hq^x dG / \int hq^d dG$ .

#### A.3.2 Impact on Firm-Specific Market Size

With the aggregate demand changes in hand, we can discuss firm responses. From the  $q^d(c) + q^x(c)$  condition, percentage change in total quantity of a firm is  $(dz(c)/dt^*)/2$ . The change in size factor is  $d\ln z(c)/dt^* = 2(1 - \theta)(d\theta/dt^*)/(1 + \theta)(1 + \theta^2)$ . Differentiating the export to domestic sales ratio,

$$\frac{d\ln\theta}{dt^*} = -\frac{1}{a^* - t^* - c(\omega)} - \frac{\eta + 2\kappa hq^x}{2(\delta + \gamma h + \kappa hq^x)q^x} \frac{dQ_i^*}{dt^*} + \frac{\eta + 2\kappa hq^d}{2(\delta + \gamma h + \kappa hq^d)q^d} \frac{dQ_i}{dt^*} < 0$$

The export-to-domestic sales ratio and hence the size factor increase with a fall in foreign tariffs.

#### A.3.3 Impact on Process Innovation

I examine how  $c_{0\omega}$  changes for exporters using the cutoff condition  $\Pi(c_{0\omega}) = \Pi(c_{0\omega} - \omega(c_{0\omega}))$ . From the envelope theorem, the derivative of the profit function at the cutoff is  $d\Pi/dt^* = \Pi_c(dc_{0\omega}/dt^*) +$

$\Pi_{t^*}$ . Let  $\Pi^\omega$  denote the profit from process innovation while  $\Pi^0$  denote the profit without prices innovation. Therefore, the change in the cost cutoff is  $[\Pi_c^\omega - \Pi_c^0] (dc_{0\omega}/dt^*) = \Pi_{t^*}^\omega - \Pi_{t^*}^0$ .

As  $\theta(c - \omega(c)) \geq \theta(c)$  and  $\Pi^\omega = \Pi^0$  at the cutoff,  $(1 + \theta_\omega^2) (hq^d)_\omega^2 = (1 + \theta_0^2) (hq^d)_0^2$  implying  $(hq^d)_\omega^2 / (hq^d)_0^2 = (1 + \theta_0^2) / (1 + \theta_\omega^2) < 1$ . Further,  $(hq^x)_\omega^2 / (hq^x)_0^2 = \theta_\omega^2 (hq^d)_\omega^2 / \theta_0^2 (hq^d)_0^2 = [\theta_\omega^2 / (1 + \theta_\omega^2)] / [\theta_0^2 / (1 + \theta_0^2)] \geq 1$  as  $\theta^2 / (1 + \theta^2)$  is increasing in  $\theta$ . Let  $y \equiv q^d + q^x$ . Then  $(hy)_\omega^2 \geq (hy)_0^2$  because  $(1 + \theta_\omega)^2 / (1 + \theta_\omega^2) \geq (1 + \theta_0)^2 / (1 + \theta_0^2)$  as  $(1 + \theta)^2 / (1 + \theta^2)$  is increasing in  $\theta$ . From the profit function,  $\Pi_{t^*} = -(dQ_i/dt^*) hq^d (\eta + \kappa hq^d) - (dQ_i^*/dt^*) hq^x (\eta + \kappa hq^x) - hq^x (\eta + \kappa hq^x)$ . As  $\eta + 2\kappa hq^d > 0$ ,  $hq^d (\eta + \kappa hq^d)$  is increasing in  $hq^d$  and similarly for  $hq^x$ . From these relationships, we find the RHS is positive.

For the LHS, we can use the envelope theorem to get  $\Pi_c^\omega = (-1 + \omega'(c)) (hy)_\omega$  and  $\Pi_c^0 = -(hy)_0$ . As  $(hy)_\omega > (hy)_0$  and  $\omega'(c) \leq 0$ , the term in square brackets on the LHS is  $-[(hy)_\omega - (hy)_0] + \omega'(c) (hy)_\omega < 0$ . Therefore,  $dc_{0\omega}/dt^* < 0$  and a fall in foreign tariffs makes it easier to undertake process innovation.

With a bilateral reduction in tariffs, the RHS is  $[(hy(\eta + \kappa hy))_\omega - (hy(\eta + \kappa hy))_0] (dQ_i/dt^*) + (hq^x)_\omega - (hq^x)_0 > 0$  and results are similar to the earlier case. A unilateral home tariff reduction has the opposite effects as  $\Pi_{t^*}^* = -(dQ_i^*/dt^*) hq^d (\eta + \kappa hq^d) - (dQ_i/dt^*) hq^x (\eta + \kappa hq^x) < 0$ .

### A.3.4 Impact on Product Innovation

From the product FOC,  $\Pi = [\gamma + \kappa Q_i + \theta^2 (\gamma + \kappa Q_i^*)] (hy)^2 / (1 + \theta)^2$  implying  $2\Pi (d \ln h / dt^*) = \Pi_{t^*} - \kappa (hq^d)^2 (1 - \theta^2 A^x / A^d) (dQ_i / dt^*)$ . Substituting for  $\Pi_{t^*}$ , the change in product range can be written as

$$-\frac{2\Pi}{h^2 q^d} \frac{dh}{dt^*} = \theta + \left[ \eta \left( 1 - \theta A^x / A^d \right) + 2\kappa hq^d \left( 1 - \theta^2 A^x / A^d \right) \right] (dQ_i / dt^*)$$

As  $\eta + 2\kappa hq^d (1 + \theta) > 0$ , the term in square brackets is positive. For a non-exporter,  $\theta = 0$  and  $\eta + 2\kappa hq^d > 0$ . Therefore,  $dh/dt^* > 0$  implying non-exporters reduce product innovation with a reduction in foreign tariffs. For exporters, the RHS is decreasing in  $c$  because  $\theta'(c) < 0$  and the term in square brackets is increasing in  $c$  (see online Appendix for details). For low cost exporters,  $dh/dt^*$  keeps falling implying they engage in greater product innovation than non-exporters. As long as  $\theta(0) \geq A^x / A^d$ , the lowest cost firms have  $dh/dt^* < 0$ . When  $\kappa = 0$ ,  $A^x / A^d = \int hq^x dG / \int hq^d dG$  which is the average export to domestic sales ratio in the economy.

### A.3.5 Aggregate Productivity Growth

Output per worker rises with selection and process innovation. This can be seen most easily for a move from autarky to free trade. Output per worker is

$$\bar{\Phi} = \frac{\int_0^{c_d} [c_d - c + 1_{c \leq c_{0\omega}} (\omega(c) - \omega(c_{0\omega}))] dG}{\int_0^{c_d} (c - 1_{c \leq c_{0\omega}} \omega(c)) [c_d - c + 1_{c \leq c_{0\omega}} (\omega(c) - \omega(c_{0\omega}))] dG}$$

Change in output per worker is

$$\begin{aligned} \frac{d \ln \bar{\Phi}}{d \ln L} = & \frac{c_d \int_0^{c_d} [1 - \bar{\Phi}(c - 1_{c \leq c_{0\omega}} \omega(c))] dG}{\int_0^{c_d} [c_d - c + 1_{\omega > 0}(\omega(c) - \omega(c_{0\omega}))] dG} \frac{d \ln c_d}{d \ln L} \\ & + \frac{-\omega'(c_{0\omega}) c_{0\omega} \int_0^{c_{0\omega}} [1 - \bar{\Phi}(c - \omega(c))] dG}{\int_0^{c_d} [c_d - c + 1_{\omega > 0}(\omega(c) - \omega(c_{0\omega}))] dG} \frac{d \ln c_{0\omega}}{d \ln L} \end{aligned}$$

I show that the coefficient on  $d \ln c_d / d \ln L$  is negative and on  $d \ln c_{0\omega} / d \ln L$  is positive so selection and process innovation increase average productivity. The sign of each coefficient can be determined as follows. The denominator on the RHS is positive as it is simply the unweighted average cost across all firms. The sign of the numerator is negative for  $d \ln c_d / d \ln L$  because  $\int_0^{c_d} \bar{\Phi}(c - 1_{c \leq c_{0\omega}} \omega(c)) dG \geq 1$  and positive for  $d \ln c_{0\omega} / d \ln L$  because  $\int_0^{c_{0\omega}} \bar{\Phi}(c - \omega(c)) dG \leq 1$ . To determine these signs, let  $\tilde{x} = \int x(c) dG$  for brevity. Then  $\int_0^{c_d} \bar{\Phi}(c - 1_{c \leq c_{0\omega}} \omega(c)) dG = \widetilde{c(\omega)} \cdot \widetilde{hq} / \widetilde{c(\omega)} \widetilde{hq}$ . Substituting for  $hq$ ,  $\widetilde{c(\omega)} \cdot \widetilde{hq} = \widetilde{c(\omega)} \cdot [c_d - \widetilde{c(\omega)}] / 2(\gamma + \kappa Q_i)$  and  $\widetilde{c(\omega)} \widetilde{hq} = [c(\omega) c_d - c(\omega)^2] / 2(\gamma + \kappa Q_i)$ . As  $\widetilde{c(\omega)}^2 \geq c(\omega)^2$ , the numerator for  $d \ln c_d / d \ln L$  is negative. The second numerator is positive if  $(\int_0^{c_{0\omega}} c(\omega) dG) \cdot \widetilde{hq} \leq \widetilde{c(\omega)} \widetilde{hq}$ . Substituting for  $hq$ , this is true if  $(\int_0^{c_{0\omega}} c(\omega) dG) [c_d - \widetilde{c(\omega)}] - [c(\omega) c_d - c(\omega)^2] \leq 0$ . Expanding the bounds of integration, the LHS of the previous expression is  $(\int_0^{c_{0\omega}} c(\omega) (c(\omega) - \widetilde{c(\omega)}) dG) - [\int_{c_{0\omega}}^{c_d} c(c_d - c) dG]$ . As  $c(\omega) \leq c_{0\omega}$  for  $c \leq c_{0\omega}$ , the LHS is less than  $c_{0\omega} (\int_0^{c_{0\omega}} c(\omega) dG - \widetilde{c(\omega)}) - [\int_{c_{0\omega}}^{c_d} c(c_d - c) dG]$ . The first term of this expression is negative as the average cost of process innovators is lower than the average cost of all producers. The second term is positive for all producers so the LHS is negative as expected.